1. QUESTION

In this question we consider the line l through the points $(-1,2)$ and $(3,10)$.

[2] (a) Compute the slope of the line l.

ANSWER

The slope is $\frac{y_2-y_1}{x_2-x_1} = \frac{10-2}{3--1} = \frac{8}{4} = 2.$

[4] (b) Give the slope-intercept form of the line l.

ANSWER Slope intercept form is $y = \underset{\uparrow \text{ slope}}{m}x + b$

$(-1,2)$ is on the line.

The line is $y - y_0 = m(x - x_0)$

$$y-2 = 2(x--1)$$
$$y-2 = 2x+2$$
$$y = 2x+4.$$

2. QUESTION

[2] (a) Determine the slope of any line that is perpendicular to the line $y = \frac{3}{4}x+5$. Motivate your answer briefly.

ANSWER

Two Perpendicular lines with slopes m and n, respectively satisfy $m \cdot n = -1$

The slope of any line that is perpendicular to $y = \frac{3}{4}x+5$ is thus $m = \frac{-1}{3/4} = -\frac{4}{3}$.

3. QUESTION

[4] (a) Determine the values of a such that the distance between the points $(-2,7)$ and $(1,a)$ is 5.

ANSWER The distance formula is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Here :

$$\sqrt{(-2-1)^2+(7-a)^2} = 5$$

$$\sqrt{3^2+(7-a)^2} = 5$$

$$3^2+(7-a)^2 = 5^2$$

$$(7-a)^2 = 5^2-3^2 = 25-9 = 16$$

$(7-a) = 4$ or $(7-a) = -4$

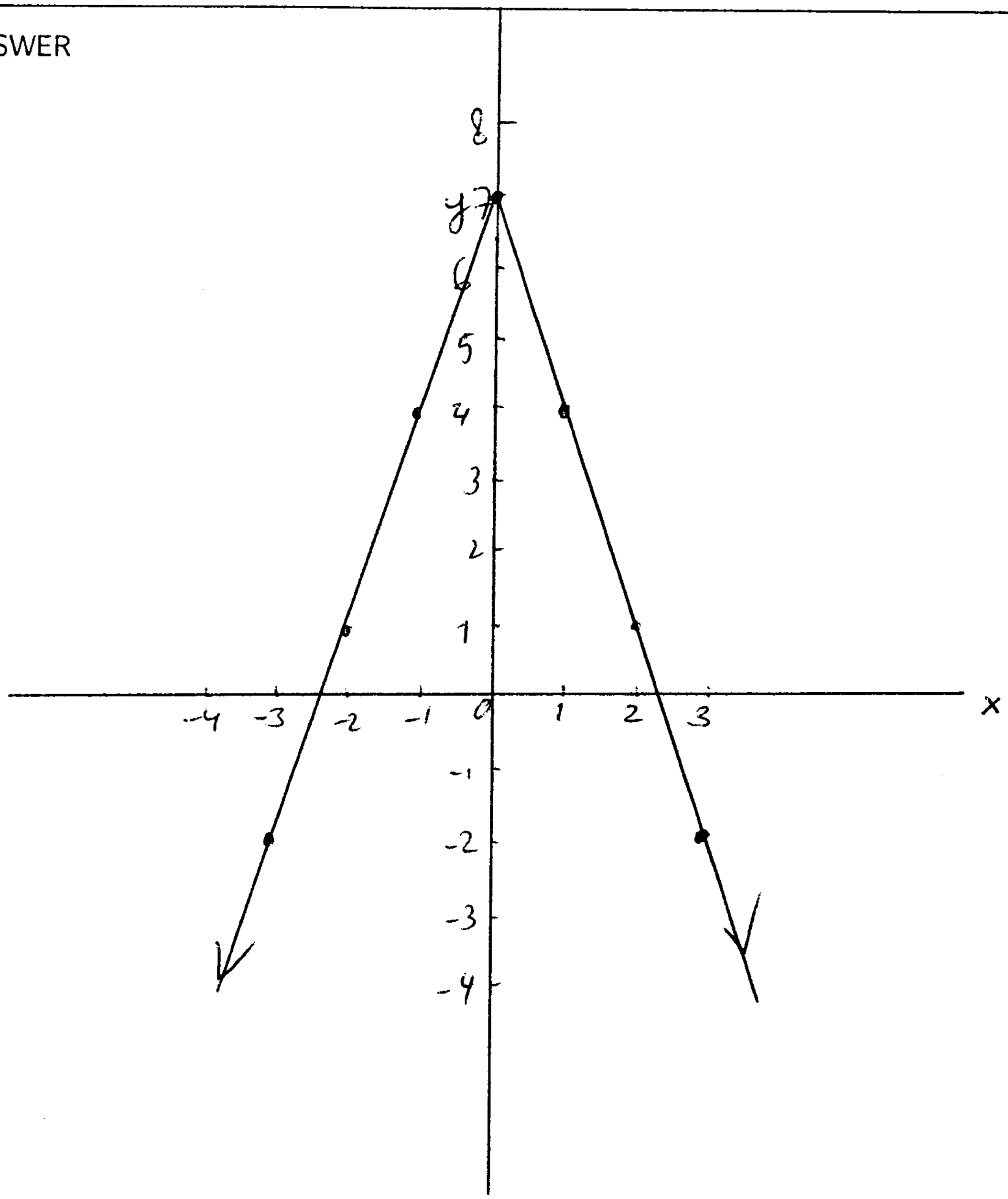
$a = 3$ or $a = 11$

4. QUESTION

In this question we consider the function f given by $f(x) = -3|x| + 7$.

[4] (a) Draw the graph of f.

ANSWER



[1] (b) Use the graph of f to determine the number(s), if any, at which f has a relative minimum or maximum. What are these minima and/or maxima?

ANSWER

f has a relative maximum at $x=0$.
The maximum is 7.

It has no other minima or maxima

[1] (c) Determine if f is odd, even or neither. Motivate your answer.

ANSWER f is even because

1) $f(-x) = -3|-x|+7$
$= -3|x|+7$
$= f(x)$

or 2) the graph is symmetric about the y-axis.

[2] (d) Solve the inequality $-3|x| + 7 \geq -2$.

ANSWER

1) $-3|x|+7 \geq -2$
$-3|x| \geq -9$ ← swap the inequality sign
$|x| \leq +3$
$-3 \leq x \leq 3$
The solution set is $\{x \mid -3 \leq x \leq 3\}$ or $[-3,3]$

or 2) Look in the graph and determine all x-values for which $f(x) \geq -2$. This gives the same answer

5. QUESTION

[4] (a) Write the circle given by the equation

$$x^2 + y^2 - 4x + 2y - 2 = 0$$

in standard form.

ANSWER

$x^2+y^2-4x+2y-2 = 0$

$x^2-4x + y^2+2y - 2 = 0$

complete squares.

$(x^2-4x+4)-4 + (y^2+2y+1)-1-2=0$

$(x-2)^2+(y+1)^2-4-1-2 = 0$

$(x-2)^2+(y+1)^2 = 7.$

$\left((x-2)^2 + (y-(-1))^2 = (\sqrt{7})^2 \right).$

[2] (b) What is the center of the circle given by $(x + 2)^2 + (y - 3)^2 = 4$?

ANSWER

$(x+2)^2 + (y-3)^2 = 4$

$(x-(-2))^2+(y-3)^2 = 2^2$

The center is $(-2,3)$.

6. QUESTION

In this question the functions f and g are given by

$$f(x) = -\frac{2}{x} + 8 \quad \text{and} \quad g(x) = \frac{-2}{x-8}.$$

[2] (a) Verify that $(f \circ g)(x) = x$, for $x \neq 8$.

ANSWER

$$(f \circ g)(x) = f(g(x)) \qquad (x \neq 8)$$
$$= f\left(\frac{-2}{x-8}\right)$$
$$= \frac{-2}{\left(\frac{-2}{x-8}\right)} + 8$$
$$= \frac{(-2)}{1} \cdot \left(\frac{x-8}{-2}\right) + 8 = x - 8 + 8 = x.$$

[2] (b) Given that also $(g \circ f)(x) = x$ for $x \neq 0$, what is special about f and g?

ANSWER They are each others inverses.

or

g is the inverse of f.

or

f is the inverse of g.