

Math 100 — First Midterm

Instructor: Dr. Sandy Rutherford

Date: October 6, 2004

Time: 11:30 – 12:30

Name (please print):_____

Signature:_____

Student Number:_____

- Read questions carefully, show all of your work and present organised solutions. If we do not understand what you are doing, you will receive *zero marks*. You *must* give reasons for your answers.
- No calculators of *any* kind may be used.
- No notes or textbooks may be used.
- Some answers may contain radicals of the form $\sqrt{2}$. These should be simplified where possible; however, a decimal expansion is not required.
- This exam has 7 pages, including this cover page. Please be sure that you have all of the pages.

Question	Mark	Total Marks
1		3
2		6
3		6
4		6
5		18
6		6
Total		45

1. State the definitions of a rational and an irrational number.

2. Solve for x in the following equations.

(a) $\frac{4}{x+3} + x = 2$

(b) $x^2 = \frac{2}{x^2 - 2}$

3. For each of the following polynomials, state if they are prime or may be factored into polynomials with integer coefficients. It is unnecessary to give an explicit factorisation. However, in each case you *must* support your answer either by giving the factorisation or by explaining your reasoning.

(a) $f(x) = 4x^2 + 1$

(b) $g(x) = x^5 - x^2 + x + 3$

4. Consider the cubic equation

$$x^3 + x^2 + ax = 0, \quad \text{where } a \text{ is a real number.}$$

Determine for which values of a this equation has 0,1,2, or 3 real solutions. Can it ever have more than 3 solutions? Explain.

5. State the domain and range of each of the following functions. Also, find any roots (values of x at which the function is equal to zero) and state which functions are even or odd under reflection about the y -axis. Finally, sketch a reasonably accurate graph for each function.

(a) $p(x) = x^4 + 2x^2 - 8$

(b) $q(x) = \frac{x^2 + \frac{1}{x}}{x + \frac{1}{x^2}} + 2$

For this function, state the domain and range both before and after any continuous extension.

(c) $r(x) = (x - \text{int}(x))^2 - 1,$

where int is the “integer part of” function defined as $\text{int}(x)$ is the largest integer which is less than or equal to x .

6. A cell phone is situated on a straight line between two transmitting towers that are 10km apart. The received signal strength in watts from the first tower is 6 divided by the square of distance from the tower in km. The signal strength in watts from the second tower is 4 divided by the square of the distance from this tower in km. As the cell phone travels along the line from the first tower to the second, it will switch over to receiving signals from the second tower when the signal strength from the second tower equals and then exceeds the signal strength from the first tower. At what point on the line between the two towers will this occur?