Simon Fraser University
Math 100

Midterm 1- Exam 1

Last Name (print): Solutions. **First Name** _____

Signature: _____ **SFU Email ID:** _____

Instructions:

1. Do not open this exam until instructed to do so.
2. **No calculators, notes or books are allowed.**
3. When presenting a final answer for your solution, calculator-ready expressions will be given full credit.
4. Show all your work. **No credit** will be given for an answer without the correct accompanying work.
5. Answer the questions in the space provided. Continue on the back of the previous page if necessary.

Question	Mark	Maximum
1		8
2		8
3		10
4		11
5		8
6		5
Total		50

1. Consider the points $A(-3,-1)$ and $B(-1,5)$:

[2 pts] (a) Find the midpoint P of the segment with endpoints A and B.

$$M = \left(\frac{-3-1}{2}, \frac{-1+5}{2}\right) = (-2, 2)$$

[2 pts] (b) Determine the distance between A and B.

$$d = \sqrt{(-1+3)^2 + (5+1)^2} = \sqrt{4+36} = \sqrt{40}$$

[4 pts] (c) Give the equation of the circle with *diameter* the segment from A to B.

$C = (-2, 2)$.

$r = \frac{\sqrt{40}}{2}$

$(x+2)^2 + (y-2)^2 = \frac{40}{4}$

$(x+2)^2 + (y-2)^2 = 10$.

2. This question deals with lines and their equations.

[2 pts] (a) Find the equation of the line L with slope $m = 4$ and x-intercept $(-2, 0)$.

$y - 0 = 4(x+2)$

$y = 4x + 8$

or

$y = 4x + b$

$0 = 4(-2) + b$

$b = 8$

$y = 4x + 8.$

[3 pts] (b) Find the equation of the line L_2 perpendicular to L passing through the point $(-1, 2)$.

Slope L_2 : $-1/4$

$y - 2 = -\frac{1}{4}(x+1)$

$y = -\frac{1}{4}x - \frac{1}{4} + 2$

$y = -\frac{1}{4}x + 7/4$

or

$y = -\frac{1}{4}x + b$

$2 = -\frac{1}{4}(-1) + b$

$2 = \frac{1}{4} = b$

$b = 7/4$

(c) Find the intersection of the lines with equation: $y = 3x+8$ and $y = -2x+3$.

[3 pts]

We require:

$3x + 8 = -2x + 3$

$5x = 3 - 8$

$x = -1$

$y = -2(-1) + 3$

$y = 5$

Point $(-1, 5)$

3. Consider the function $f(x) = x^2$.

[2 pts] (a) Find the vertex of the parabola given by $g(x) = f(x + \sqrt{2}) - 2$.

$V = (-\sqrt{2} \ -2)$ $\quad g(x) = (x+\sqrt{2})^2 - 2$

The graph of g is a hor. shift & vert. shift of the graph of f.

[2 pts] (b) Give the equation of the axis of symmetry of the parabola defined by $h(x) = -3f(x-4)$.

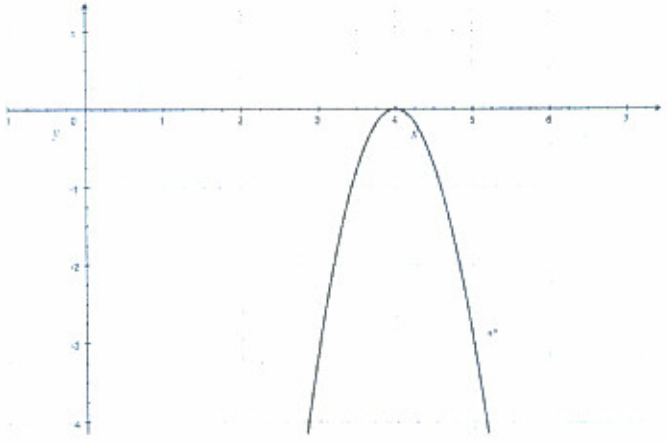
$h(x) = -3(x-4)^2$

$x = 4$

[2 pts] (c) Which (if any) of $f(x)$, $g(x)$ and $h(x)$ is an even function? Explain

Only $f(x)$ is even. $f(-x) = (-x)^2 = x^2 = f(x)$

[4 pts] (d) Sketch the graph of $h(x)$.



4. Consider the functions $f(x) = \frac{2}{5}x - 4$, $g(x) = \sqrt{x^2 - 9}$ and $h(x) = \dfrac{\sqrt{x^2-9}}{x-3}$.

[3 pts] (a) Find the equation of $f^{-1}(x)$.

$y = \frac{2}{5}x - 4$ $\qquad$ $y = \frac{5}{2}x + 10$

$y + 4 = \frac{2}{5}x$ $\qquad$ $f^{-1}(x) = \frac{5}{2}x + 10$

$\frac{5}{2}(y+4) = x$

$\frac{5}{2}y + 10 = x$

[2 pts] (b) Find an expression for $(f \circ g)(x)$.

$f(g(x)) = f(\sqrt{x^2-9}) = \frac{2}{5}(\sqrt{x^2-9}) - 4.$

[3 pts] (c) Find an expression for $(gh)(x)$ **and simplify**.

$gh(x) = \sqrt{x^2-9}\ \dfrac{\sqrt{x^2-9}}{x-3}$

$= \dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{x-3} = x+3.$

[3 pts] (d) Determine the domain of $(gh)(x)$

Dom. g : $x^2 - 9 \geq 0$ or $|x| \geq 3$ $\qquad (-\infty, -3] \cup [3, \infty)$

Dom h. also requires $x - 3 \neq 0$ or $x \neq 3$.

Dom. of $gh(x)$: $\qquad (-\infty, -3] \cup (3, \infty)$

5. You have 40 m of fencing to enclose a rectangular region.

[4 pts] (a) Give an expression for the area of the region as a function of the length of one side of the rectangle.

Let x & y be the dimensions

Area: xy

But $2x + 2y = 40$

$2(x+y) = 40$

$x + y = 20$

$y = 20 - x.$

$$\text{Area} = x(20-x) = 20x - x^2$$

[3 pts] (b) Find the dimensions of the rectangle that maximize the enclosed area.

The area function is quadratic, so we need to find the vertex.

$$A(x) = -x^2 + 20x$$
$$= -(x^2 - 20x + 100 - 100)$$
$$= -(x-10)^2 + 100$$

Thus $x = 10$

$y = 20 - 10$

$y = 10$

[1 pts] (c) What is the maximum area?

$A = 100.$

6. Consider the function:

$$h(x) = \begin{cases} 3x^2 + 2 & \text{if } x < 3 \\ x - 5 & \text{if } 3 < x \leq 20 \end{cases}$$

[1 pts] (a) Evaluate $h(5)$.

$h(5) = 5 - 5 = 0$

[2 pts] (b) Give the domain of $h(x)$.

$(-\infty, 3) \cup (3, 20]$

[2 pts] (c) What is the range of $|h(x)|$?

Range is $[0, \infty)$