

Simon Fraser University

Math 100

Midterm 1

Instructor : Sue Habberger

Date: February 7, 2007

Time: 11:30 - 12:20am

Last Name (print): _____

KEY

First Name: _____

Signature: _____

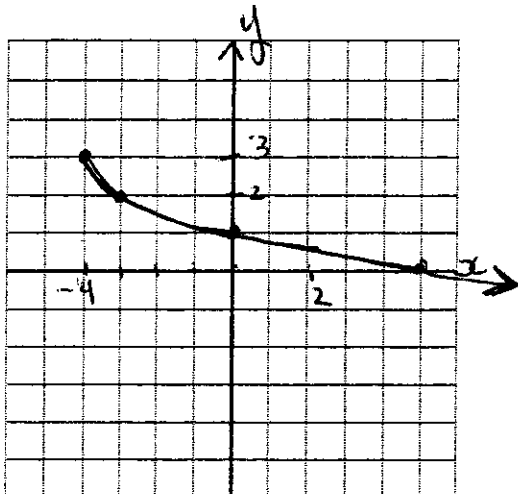
SFU Email ID: _____

Instructions:

- 1. Do not open this exam until instructed to do so.**
- 2. Ensure that you have 5 pages of questions numbered page 2 to page 6.**
- 3. No calculators, notes or books are allowed.**
- 4. Give all final numerical answers exactly, simplify all final expressions.**
- 5. For full marks, show all steps leading to your final answer.**
- 6. Answer each question in the space provided. Continue on the back of the previous page if necessary.**

Question	Mark	Maximum
1		5
2		6
3		6
4		8
5		5
TOTAL		30

1. a) (3 marks) Draw a careful graph of the function: $y = -\sqrt{x+4} + 3$



$$y = \sqrt{x}$$
$$y = \sqrt{x+4}$$

$$y = -\sqrt{x+4}$$

$$y = -\sqrt{x+4} + 3$$

4 left.
vertical flip.

3 up

- b) (2 marks) State the Domain and Range of the function

DOMAIN: $[-4, \infty)$

OR $x \geq -4$

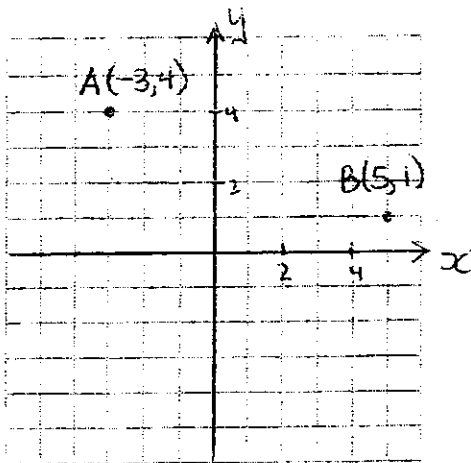
OR $\{x \mid x \geq -4\}$

RANGE: $(-\infty, 3]$

OR $y \leq 3$

OR $\{y \mid y \leq 3\}$

2. Given two points: $A(-3,4)$ and $B(5,1)$ as shown on the graph:



a) (1 mark) Determine the co-ordinates of the midpoint of \overline{AB}

$$\left(\frac{-3+5}{2}, \frac{4+1}{2} \right) = \left(1, \frac{5}{2} \right)$$

b) (1 mark) Determine the slope of \overline{AB}

$$\frac{4-1}{-3-5} = -\frac{3}{8}$$

c) (1 mark) Determine the equation of the line containing A and B

$$(y-4) = -\frac{3}{8}(x+3) \quad \text{or} \quad (y-1) = -\frac{3}{8}(x-5)$$

d) (1 mark) Determine the equation of the vertical line through B

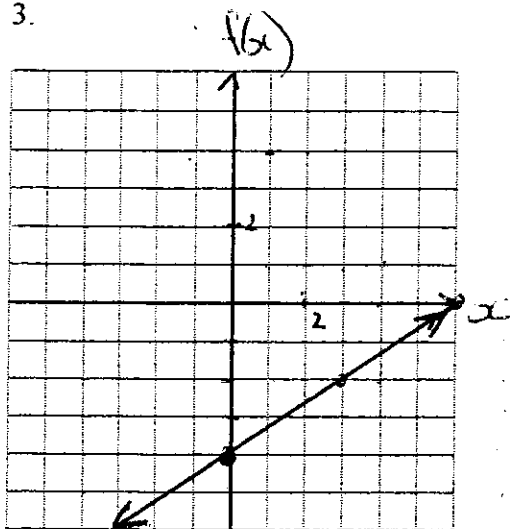
$$x = 5$$

e) (2 marks) Determine the equation of a circle, centre A , passing through B

$$\text{RADIUS: } r^2 = (5+3)^2 + (1-4)^2 = 64+9 = 73 \\ (r = \sqrt{73})$$

$$\text{EQUATION: } (x+3)^2 + (y-4)^2 = 73$$

3.



a) (2 marks) Draw a careful graph of the linear function:

$$f(x) = \frac{2}{3}x - 4$$

Slope: $\frac{2}{3}$

intercept: $y = -4$

b) (2 marks) Determine the equation of a line perpendicular to the graph of $f(x)$ passing through the point $(7, -5)$

Slope: $-\frac{3}{2}$

$$(y + 5) = -\frac{3}{2}(x - 7)$$

OR $y = -\frac{3}{2}x + \frac{11}{2}$

c) (2 marks) Determine a formula for $f^{-1}(x)$, the inverse of $f(x)$

$$y = \frac{2}{3}x - 4 \Rightarrow x = \frac{3}{2}y + 6$$

$$3x = 2y + 12$$

$$3x + 12 = 2y$$

$$\frac{3}{2}x + 6 = y$$

$$f^{-1}(x) = \frac{3}{2}x + 6$$

4. Given two functions: $f(x) = \frac{\sqrt{1+x}}{2-x}$ and $g(x) = 3-x^2$

a) (1 mark) Evaluate $f(8)$

$$= \frac{\sqrt{9}}{(2-8)} = \frac{3}{-6} = -\frac{1}{2}$$

b) (1 mark) Evaluate $(fg)(0)$

$$= f(0) \cdot g(0)$$

$$= \frac{\sqrt{1}}{2} \cdot 3 = \frac{3}{2}$$

c) (1 mark) Give a simplified expression for $g(x+h)$

$$= 3 - (x+h)^2$$

← either one →

$$= 3 - (x^2 + 2xh + h^2) = 3 - x^2 - 2xh - h^2$$

d) (2 marks) Give a simplified expression for $(f \circ g)(x)$

$$= f(g(x)) = f(3-x^2) = \frac{\sqrt{1+(3-x^2)}}{2-(3-x^2)}$$

$$= \frac{\sqrt{4-x^2}}{x^2-1}$$

e) (3 marks) Solve for x : $f(x) = \frac{1}{2}$

$$\frac{\sqrt{1+x}}{2-x} = \frac{1}{2} \Rightarrow \left[\frac{\sqrt{1+x}}{2-x} \right]^2 = \left[\frac{1}{2} \right]^2$$

$$4(1+x) = 4 - 4x + x^2$$

$$4 + 4x = 4 - 4x + x^2$$

$$0 = x^2 - 8x$$

$$0 = x(x-8)$$

$$x = 0 \text{ OR } x = 8. \quad \text{But } 8 \text{ is extraneous.}$$

∴ only solution is $x = 0$

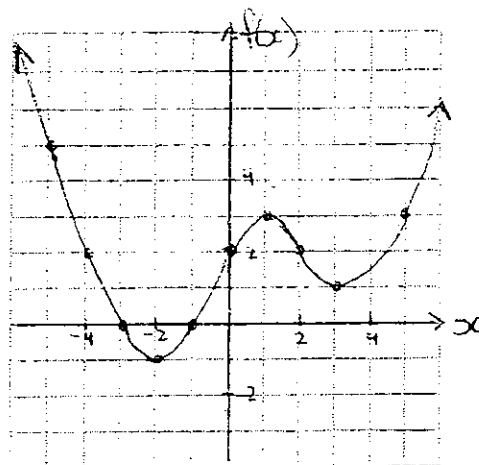
5) (5 marks – one mark per answer)

The graph of a function: $f(x)$ is shown on the grid at right.

Use this graph to determine each of the following:

a) The value of $f(5)$

3



b) The intervals on which f is decreasing

$(-\infty, -2)$ $(1, 3)$

c) The co-ordinates of any relative maximum points for $f(x)$

$(1, 3)$

d) The value(s) of x for which $f(x) = 0$

$x = -3$ or $x = -1$

e) The range of $f(x)$ (answer in interval notation)

$[-1, \infty)$