

Simon Fraser University
Math 100

Midterm 1

Date: October 4, 2006

Time: 11:30 - 12:20am

Last Name (print): Solutions First Name _____

Signature: _____ SFU Email ID: _____

Instructor: Laura Chávez Lomeli.

Instructions:

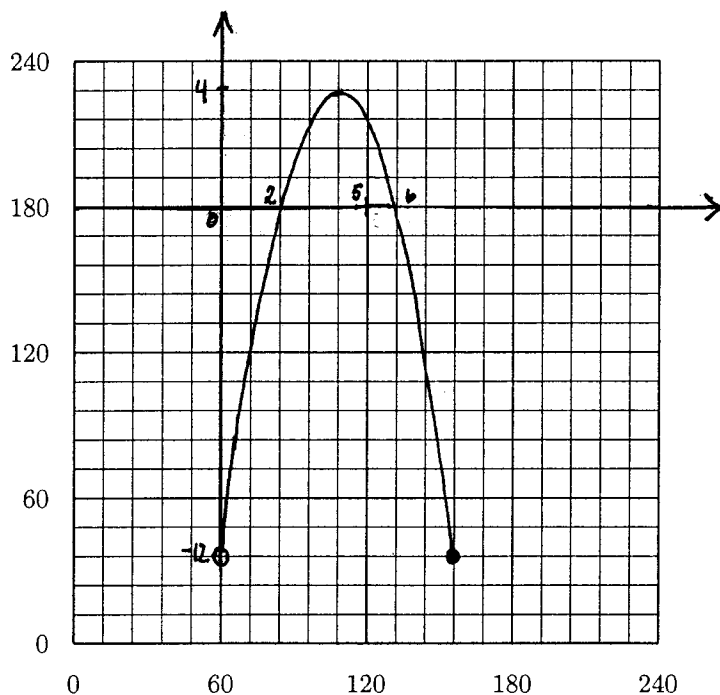
1. Do not open this exam until instructed to do so.
2. No calculators, notes or books are allowed.
3. When presenting a final answer for your solution, calculator-ready expressions will be given full credit.
4. Show all your work. **No credit** will be given for an answer without the correct accompanying work.
5. Answer the questions in the space provided. Continue on the back of the previous page if necessary.

Question	Mark	Maximum
1		10
2		8
3		4
4		9
5		9
Total		40

1. Given the function: $f(x) = -(x - 4)^2 + 4$ defined for $0 < x \leq 8$:

[6 marks]

- (a) Draw the graph of $f(x)$.



[2 marks]

- (b) Is the function one to one? Explain.

No. Horizontal line test shows this.
eg. $y=0$ intersects the curve
twice

[2 marks]

- (c) Determine the range of $f(x)$.

$-12 \leq y \leq 4$ or $[-12, 4]$

2. Let L be the line with equation $-2y + 8x - 32 = 0$.

[2 marks]

(a) Find the slope of L .

$$y - 4x + 16 = 0$$

or Find 3 pts.

$$y = -4x - 16$$

$$m = 4$$

[3 marks]

(b) Determine the x -intercept and y -intercept of L .

x -int. solve $y=0$

$$0 = 4x - 16$$

$$4x = 16$$

$$x = 4$$

y -int.

$$y = 4 \cdot 0 - 16$$

$$y = -16$$

[3 marks]

(c) Find the equation of the line perpendicular to L and with the same x -intercept.

$m = -\frac{1}{4}$ & goes through $(4, 0)$

$$y = -\frac{1}{4}x + b$$

$$0 = -\frac{1}{4} \cdot 4 + b$$

$$b = 1 \rightarrow$$

$$y = -\frac{1}{4}x + 1.$$

- [4 marks] 3. Find the equation of the inverse of $f(x) = \frac{10}{x} - 5$. Show all work.

$$y = \frac{10}{x} - 5$$

$$x \neq 0$$

$$x = \frac{10}{y} - 5$$

$$x + 5 = \frac{10}{y}$$

$$x \neq -5$$

$$y = \frac{10}{x+5}$$

4. Let $f(x) = x^2 - 4$ and $g(x) = 3x^2 + 12x + 12$

[3 marks]

(a) Find $g(f(x))$.

$$\begin{aligned} g(x^2 - 4) &= 3(x^2 - 4)^2 + 12(x^2 - 4) + 12 \\ &= 3(x^4 - 8x^2 + 16) + 12x^2 - 48 + 12 \\ &= 3x^4 - 24x^2 + 48 + 12x^2 - 48 + 12 \\ &= 3x^4 - 12x^2 + 12. \end{aligned}$$

[4 marks]

(b) Find $h(x) = \frac{f(x)}{g(x)}$ and simplify as much as possible.

$$\begin{aligned} h(x) &= \frac{x^2 - 4}{3x^2 + 12x + 12} \\ &= \frac{(x-2)(x+2)}{3(x^2 + 4x + 4)} = \frac{(x-2)(x+2)}{3(x+2)^2} = \frac{x-2}{3(x+2)} \end{aligned}$$

[2 marks]

(c) Determine the domain of $h(x)$.

$$\text{Domain of } h(x) = \{x \mid x \neq -2\}$$

$$\text{or } (-\infty, -2) \cup (-2, \infty)$$

$$\text{or } x \neq -2$$

5. Consider the points $P = (-3, 1)$, $Q = (5, 15)$ and $R = (0, -3)$.

[2 marks]

(a) Find the midpoint of the segment with endpoints **P** and **Q**.

$$\left(\frac{-3+5}{2}, \frac{1+15}{2} \right) = (1, 8)$$

[2 marks]

(b) Calculate the distance between **P** and **R**.

$$d = \sqrt{(-3-1)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

[2 marks]

(c) Determine the equation of the circle with center **Q** and radius equal to the distance between **P** and **R** (as found in part 5b).

$$(y-15)^2 + (x-5)^2 = 25$$

[3 marks]

(d) What is the standard form of the equation of the circle: $x^2 + y^2 + 6x - 8y + 8 = 0$?

$$(x^2 + 6x + 9) + (y^2 - 8y + 16) + 8 - 9 - 16 = 0$$

$$(x+3)^2 + (y-4)^2 = 17.$$