Simon Fraser University
Math 100

Final Exam
Instructor: Sue Haberger

Date: April 17, 2007
Time: 8:30 –11:30 am

Last Name (print): KEY. **First Name:** ____________

Signature: ______________________ **SFU Email ID:** ____________

Instructions:

1. Do not open this exam until instructed to do so.

2. Ensure that you have 12 pages of questions numbered page 2 to page 13.

3. No calculators, notes or books are allowed.

4. Give all final numerical answers exactly, simplify all final expressions.

5. For full marks, show all steps leading to your final answer. Clearly indicate your final answer.

6. Answer each question in the space provided. Use the back of the previous page if necessary – if you do this, clearly instruct the marker "continued on back of previous page".

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Mark															
Maximum	16	10	10	10	6	3	7	5	5	6	2	2	5	3	90

1. (2 marks each) Give the exact simplified numerical value of each expression. Write your final answer on the line provided
(Remember that expressions such as $\sqrt{5}$, $\frac{\pi}{4}$, e^3 , and $\ln 6$ are exact numerical values)

a) The value of 80^o in radians =

$80 \times \frac{\pi}{180}$

a) $\frac{4\pi}{9}$

b) $7^0 + 8^{-\frac{4}{3}} =$

$1 + \frac{1}{2^4}$

b) $\frac{17}{16}$

$(1\frac{1}{16})$

c) $\log_5 \sqrt{5} - \log_5(\frac{1}{25}) =$

$\frac{1}{2} - (-2)$

c) $\frac{5}{2}$

$(2\frac{1}{2})$ (2.5)

d) $10^{\frac{1}{2}\log 25}$ =

$10^{\log 25^{\frac{1}{2}}}$

$10^{\log 5}$

d) 5

e) $\csc\left(\frac{5\pi}{3}\right) =$ $\frac{1}{\sin\left(\frac{5\pi}{3}\right)} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$

$\left(-\frac{2\sqrt{3}}{3}\right)$

e) $\frac{-2}{\sqrt{3}}$

f) The period of the function $f(t) = \frac{3}{2}\cos(\frac{\pi}{4}t)$ is:

f) 8

$\frac{2\pi}{\frac{\pi}{4}}$

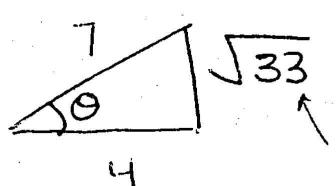
g) $\tan^{-1}(-1) =$

g) $-\frac{\pi}{4}$

h) $\cot^2(\cos^{-1}(\frac{4}{7})) =$

h) $\frac{16}{33}$

let $\theta = \cos^{-1}(\frac{4}{7})$



$\cot^2\theta = \left(\frac{4}{\sqrt{33}}\right)^2$

2. Given two functions: $f(x) = \dfrac{5}{x+1}$ and $g(x) = x^2 - 1$

a) (3 marks) Evaluate exact and simplified:

$\left(\frac{g}{f}\right)(6) = \frac{g(6)}{f(6)}$ $49.$ $= \frac{35}{\left(\frac{5}{7}\right)}$	$(gf)(0) = g(0) \cdot f(0)$ $= (-1)(5)$ $= -5$	$(g \circ f)(0) = g(f(0))$ $= g(5)$ $= 24$

b) (2 marks) Give a simplified expression for $(f \circ g)(x)$ $= f(g(x))$

$= f(x^2-1)$ $\frac{1}{2}$ $= \dfrac{5}{(x^2-1)+1} = \dfrac{5}{x^2}$.

c) (2 marks) Give a simplified expression for $\dfrac{g(3+h)-g(3)}{h}$, (assume $h \neq 0$)

$= \dfrac{(3+h)^2 - 1 - (8)}{h} = \dfrac{9+6h+h^2-1-8}{h}$

$= \dfrac{h(6+h)}{h} = \boxed{6+h}$

d) (3 marks) Determine a formula for $f^{-1}(x)$, the inverse of $f(x)$.
State the **range** of f^{-1}.

$y = \dfrac{5}{x+1}$ interchange. $x = \dfrac{5}{y+1}$

$y+1 = \dfrac{5}{x}$.

$y = \dfrac{5}{x} - 1$.

$f^{-1}(x) =$ $\left(\frac{5}{x} - 1\right)$

Range of f^{-1}: $f^{-1}(x) \neq -1$.

All real numbers except -1.

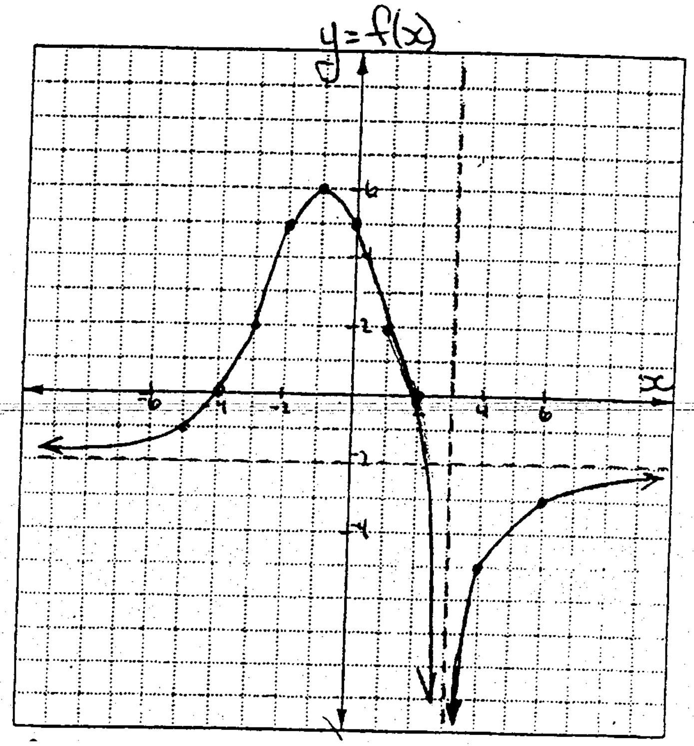
$(-\infty, -1) \cup (-1, \infty)$

Range of f^{-1} is <u>domain</u> of f

$x \neq -1$.

3. (10 marks) Use the graph of $f(x)$ shown to answer the questions:

a) $f(0) =$ 5



b) $f(-5) =$ -1

c) For what values of x is $f(x) = 0$?

$x = -4$ or 2.

d) As $x \to \infty$, $f(x) \to$ -2

e) As $x \to 3^+$, $f(x) \to$ $-\infty$

f) Give the interval where $f(x)$ is

decreasing: $(-1, 3)$

$-1 < x < 3$

g) Give the range of $f(x)$ in interval notation: $(-\infty, 6]$

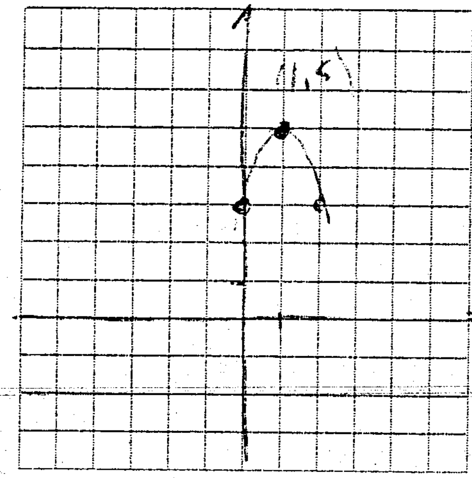
h) State the numbers (if any) at which $f(x)$ has a relative maximum:

-1

i) Solve: $f(x) \geq 0$ (answer in interval notation):

$[-4, 2]$

4. For the points: $P(-2,7)$ and $Q(1,5)$ give each of the following.
(The graph grid is for your use if needed. It will not be marked.)



a) (2 marks) The exact distance $\overline{PQ}$

$$\sqrt{(-3)^2+(2)^2} = \sqrt{13}$$

b) (1 mark) The co-ordinates of the midpoint of $\overline{PQ}$

$$\left(\frac{-2+1}{2}, \frac{7+5}{2}\right) = \left(-\frac{1}{2}, 6\right)$$

$\frac{1}{2}$

c) (1 mark) The slope of $\overline{PQ}$

$$\frac{5-7}{1-(-2)} \qquad -\frac{2}{3}.$$

d) (1 mark) An equation for the line $\overleftrightarrow{PQ}$

$$(y-5) = -\frac{2}{3}(x-1) \quad \text{OR} \quad (y-7) = -\frac{2}{3}(x+2)$$

(OR ANY EQUIVALENT) $y = -\frac{2}{3}x + \frac{17}{3}$

e) (1 mark) The equation of the vertical line through P

$$x = -2$$

f) (2 marks) The equation for a circle centre P, passing through Q.

centre $(-2,7)$ radius: $\sqrt{13}$.

$$(x+2)^2 + (y-7)^2 = 13.$$

g) (2 marks) The equation of a parabola with vertex Q, and y-intercept 3.

$$y = -2(x-1)^2 + 5$$

OR $(y-5) = -2(x-1)^2$

5. (6 marks) For the graph of the quadratic function: $f(x) = x^2 - 10x + 18$

a) Give the y-intercept

18

b) Give the co-ordinates of the vertex

$$\frac{-b}{2a} = \frac{-(-10)}{2} = 5$$

$$f(5) = 25 - 50 + 18 = -7$$

$(5, -7)$

c) Give the exact value(s) of the x- intercepts.

solve $x^2 - 10x + 18 = 0$

$$x = \frac{10 \pm \sqrt{100-72}}{2} = \frac{10 \pm \sqrt{28}}{2}$$

$(= 5 \pm \sqrt{7})$ $(5+\sqrt{7}$ & $5-\sqrt{7})$

d) Give the range in interval notation.

$[-7, \infty)$

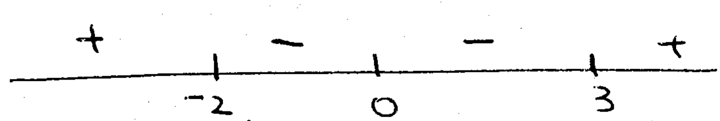
6. (3 marks) Use sign analysis on the number line to solve the inequality:

$$x^4 > x^3 + 6x^2$$

$$x^4 - x^3 - 6x^2 > 0$$

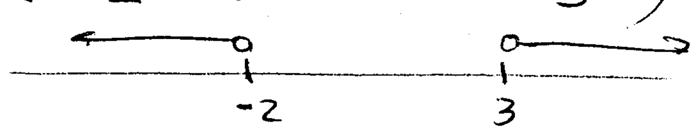
$$x^2(x^2 - x - 6) > 0$$

$$x^2(x-3)(x+2) > 0$$



$(-\infty, -2) \cup (3, \infty)$

$(x < -2$ OR $x > 3)$



7. For the polynomial function: $f(x) = 3x^3 - x^2 - 15x + 5$

a) (3 marks) State the quotient and remainder when $f(x)$ is divided by $(x + 2)$

$$\begin{array}{r|rrrr} -2 & 3 & -1 & -15 & 5 \\ & & -6 & 14 & 2 \\ \hline & 3 & -7 & -1 & 7 \end{array}$$

QUOTIENT: $3x^2 - 7x - 1$

REMAINDER: 7

b) (3 marks) Given that $\frac{1}{3}$ is a zero of $f(x)$ determine all the real zeros.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -15 & 5 \\ & & 1 & 0 & -5 \\ \hline & 3 & 0 & -15 & 0 \end{array}$$

Depressed equation: $3x^2 - 15 = 0$

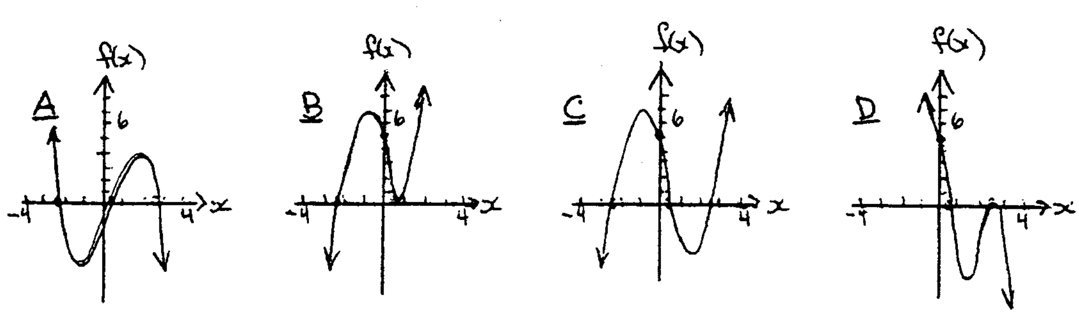
$x^2 = 5$

$x = \pm\sqrt{5}$.

All real zeros: $\left(\frac{1}{3}\right)$, $\sqrt{5}$, $-\sqrt{5}$.

c) (1 mark) Which of the following could be the graph of $f(x)$?

Write the letter of your choice: C

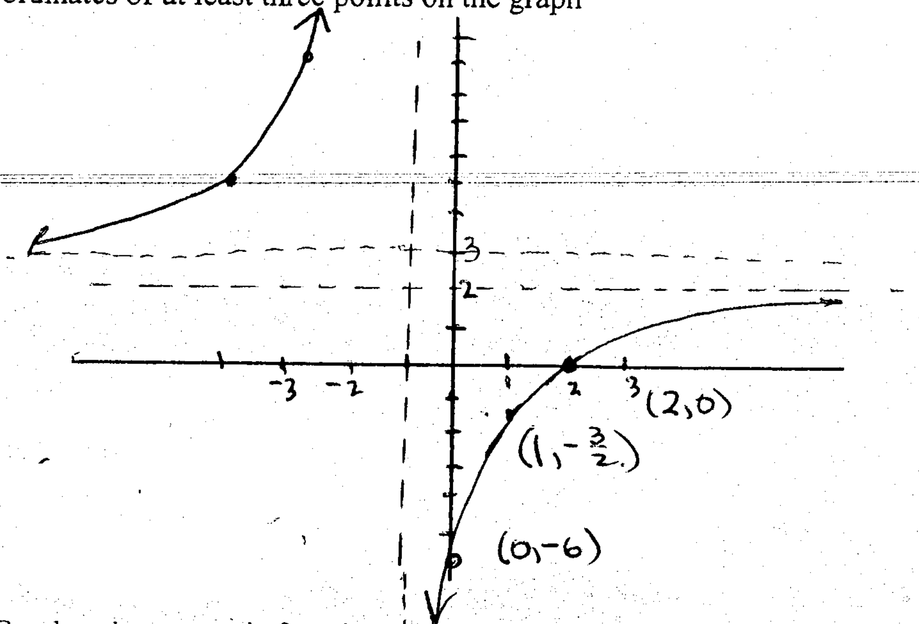


8. For the rational function: $R(x) = \dfrac{3x-6}{x+1}$ $= \dfrac{3(x-2)}{(x+1)}$

a) (1 mark) Give the equation of the horizontal asymptote: $y = 3$

b) (1mark) Give the value of any x-intercepts: 2
(OR $x=2$)

c) (3 marks) Sketch a graph of the function. Show asymptotes as dotted lines. Give the co-ordinates of at least three points on the graph



x	$R(x)$
-2	12
-3	7.5
-4	6
3	3/4
5	3/2
1	-3/2
4	6/5

↑ OTHER POINTS

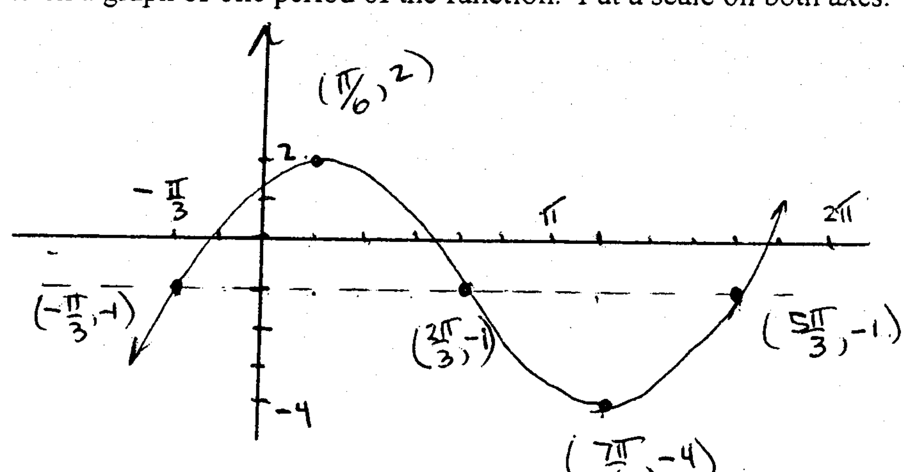
8	2

9. For the trigonometric function: $y = 3\sin(x + \frac{\pi}{3}) - 1$

a) (1 mark) Give the value and direction of the phase shift. $\frac{\pi}{3}$ LEFT (←)

b) (1 mark) Give the range of the function in interval notation. $[-4, 2]$

c) (3 marks) Sketch a graph of one period of the function. Put a scale on both axes.



10. Solve each equation. Give all solutions in exact simplified form

a) (2 marks) $3 + \log_2(x-3) = \log_2(x+11)$

$$3 = \log_2\left[\frac{x+11}{x-3}\right].$$

$$2^3 = \frac{x+11}{x-3}$$

$$8x - 24 = x + 11$$

$$7x = 35$$

$$x = 5$$

b) (2 marks) $3e^{(1-2x)} = 10$

$$e^{1-2x} = \frac{10}{3}$$

$$(1-2x)\cancel{\ln e}^{1} = \ln\left(\frac{10}{3}\right)$$

$$1 - \ln\left(\frac{10}{3}\right) = 2x$$

$$x = \frac{1 - \ln\left(\frac{10}{3}\right)}{2}$$

c) (2 marks) $\ln(10 - 7x) = \frac{1}{2}$

$$10 - 7x = e^{1/2}$$

$$10 - e^{1/2} = 7x$$

$$x = \frac{10 - e^{1/2}}{7}$$

11. (2 marks) If \$20,000 is invested at an annual rate of $4\frac{1}{2}\%$ compounded continuously, how long (in years) would it take to grow to \$25,000? (Give your answer as an exact "calculator ready" expression)

$A = Pe^{rt}$

$25{,}000 = 20{,}000\, e^{.045t}$

$\frac{5}{4} = e^{.045t}$

$\ln\left(\frac{5}{4}\right) = .045t\ (\ln e)$

$t = \dfrac{\ln\left(\frac{5}{4}\right)}{0.045}$

12. (2 marks) Simplify: $\cos(x + \frac{\pi}{2}) + \sin(\pi - x) =$

$\left(\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}\right) + \left(\sin\pi \cos x - \cos\pi \sin x\right)$

$= (0 - \sin x) + (0 - (-1)\sin x)$

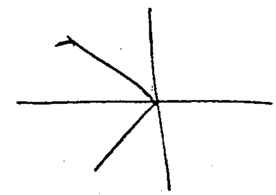
$= -\sin x + \sin x$

$= 0$

13. Solve each trigonometric equation on the interval $[0, 2\pi)$

a) (2 marks) $\sec x = -2$

$\cos x = -\frac{1}{2}$. $x_{REF}: \frac{\pi}{3}$



$x = \frac{2\pi}{3}, \frac{4\pi}{3}$.

b) (3 marks) $1 - 2\cos x = \sin^2 x + 3$

$1 - 2\cos x = (1 - \cos^2 x) + 3$

$\cos^2 x - 2\cos x - 3 = 0$

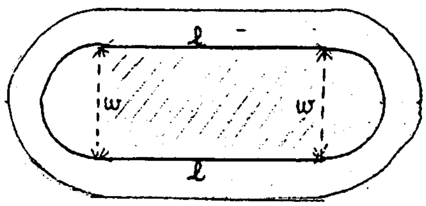
$(\cos x + 1)(\cos x - 3) = 0$

$\cos x = -1$ OR $\cos x = 3$

$\boxed{x = \pi}$ $\emptyset$

14. (3 marks) A track and field area is to be constructed in the shape of a rectangle with semicircles at each end. The inside perimeter of the track is to be 400 metres. Find the dimensions (length and width) of the rectangle that maximizes the area of the rectangular portion of the field. Give exact answers.

Note: For a circle of radius r: Circumference: $C = 2\pi r$ Area: $A = \pi r^2$



l - length

w - width.

Maximize Area = lw

$$A = lw$$

$$2l + \pi w = 400$$

$$l = \frac{400 - \pi w}{2} \qquad w = \frac{400 - 2l}{\pi}$$

$$A = \left(\frac{400 - \pi w}{2}\right) \cdot w \quad \leftarrow \checkmark \rightarrow \quad A = \frac{l(400 - 2l)}{\pi}$$

either

(VERTEX) $-\frac{b}{2a}$

(w) $\frac{-200}{2\left(-\frac{\pi}{2}\right)} = \frac{200}{\pi}$ $\qquad$ $\frac{-\frac{400}{\pi}}{2(-2)} = 100$ (l)

IF $w = \frac{200}{\pi}$, $l = \frac{400 - \cancel{\pi}\left(\frac{200}{\cancel{\pi}}\right)}{2}$

$= 100$

IF $l = 100$, $w = \frac{400 - 2(100)}{\pi} = \frac{200}{\pi}$

ANSWER: AREA IS MAXIMUM WHEN

$l = 100$ m $\quad w = \frac{200}{\pi}$ m.