MATH 100-D200 Instructor: R. Pyke

Midterm 1, October 3, 2007

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1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. Ordinary Scientific Calculators ONLY are allowed.
   NO GRAPHING CALCULATORS ALLOWED.
5. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

Question	Score	Max
1		5
2		5
3		5
4		6
5		8
6		5
7		5
8		15
Total		54

(1) [Marks: 5] Solve the following inequality. Express your answer in interval notation.

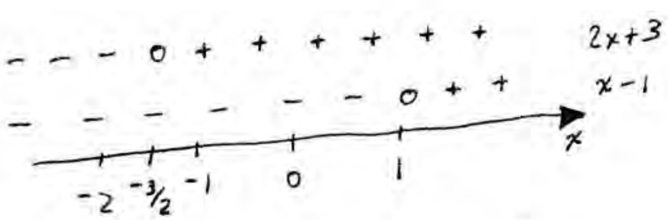
$$\left|\frac{4x+1}{x-1}\right| > 2$$

Two cases; $\dfrac{4x+1}{x-1} > 2$ or $\dfrac{4x+1}{x-1} < -2$.

$$\frac{4x+1}{x-1} > 2 \iff \frac{4x+1}{x-1} - 2 > 0$$

$$\iff \frac{4x+1-2x+2}{x-1} > 0$$

$$\iff \frac{2x+3}{x-1} > 0$$

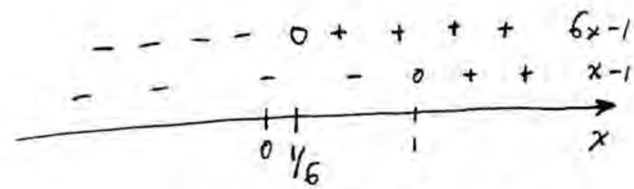


we see the allowed intervals are $\left(-\infty, -\frac{3}{2}\right) \cup (1, \infty)$

$$\frac{4x+1}{x-1} < -2 \iff \frac{4x+1}{x-1} + 2 < 0$$

$$\iff \frac{4x+1+2x-2}{x-1} < 0$$

$$\iff \frac{6x-1}{x-1} < 0$$

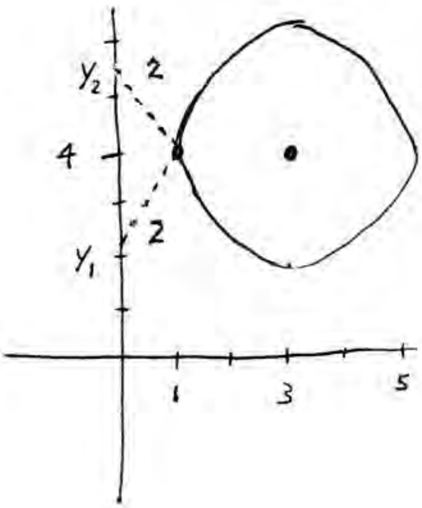


we see the allowed interval is $\left(\frac{1}{6}, 1\right)$

So solution is union of these two;

$$\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{1}{6}, 1\right) \cup (1, \infty)$$

(2) [Marks: 5] Find two points on the y-axis that are a distance of 2 units from the circle $(x-3)^2+(y-4)^2=4$ (begin by making a sketch!).



The easiest point on the circle to consider is the point nearest the y-axis; $(1,4)$. Then there are two points on the y-axis $(0,y_1)$, $(0,y_2)$ a distance 2 from $(1,4)$.

So if y is either of y_1 or y_2, then

$$4 = d^2\big((0,y),(1,4)\big)$$ distance between A,B is $d(A,B)$

$$= (0-1)^2 + (y-4)^2 = 1 + y^2 - 8y + 16$$

That is; $y^2 - 8y + 13 = 0$

using quadratic formula; $y_{1,2} = \dfrac{8 \pm \sqrt{64-52}}{2}$

(3) [Marks: 5] Find the centre and radius of the following circle;

$$12y - 16x - 2y^2 - 14 - 2x^2 = 30$$

mult. by $-\frac{1}{2}$; $\quad -6y + 8x + y^2 + 7 + x^2 = -15$

complete square $\quad (x+4)^2 + (y-3)^2 - 16 - 9 = -22$

$$\rightarrow \quad (x+4)^2 + (y-3)^2 = 3$$

Centre $(-4,3)$ radius $\sqrt{3}$

(4) [Marks: 6] Determine whether the graphs of the following equations possess symmetry with respect to the x-axis, y-axis, or origin, and find any x and y intercepts.

(a) $|3x^2y| = 4$ $\quad F(-x,-y) = \left| 3|-x|^2|-y| \right| - 4 = |3x^2y|-4$

$F(x,y) = |3x^2y|-4$ $\quad = F(x,y)$. So symmetric w.r.t. origin.

$\boxed{\text{No intercepts}}$

$\left.\begin{array}{l} F(-x,y) = F(x,y) \\ F(x,-y) = F(x,y) \end{array}\right\}$ so symmetric wrt x and y axis

(plot it!)

(b) $2x^3 - y = \dfrac{x}{y^2+1}$ $\quad F(-x,-y) = -2x^3 + y + \dfrac{x}{y^2+1} = 0$ is same as $F(x,y)=0$, so symmetric w.r.t. origin.

$F(x,y) = 2x^3 - y - \dfrac{x}{y^2+1} = 0.$

$F(-x,y)=0$ and $F(x,-y)=0$ are NOT same as $F(x,y)=0$ $\leftarrow$ But NOT symmetric with respect to x, y axis.

$x=0;\ -y=0 \rightarrow (0,0)$ y intercept

$y=0;\ 2x^3 = x \rightarrow 2x^3 - x = 0 \rightarrow x(2x^2-1)=0;\quad x=0 /\ x = \pm\frac{1}{\sqrt{2}};\quad (\frac{1}{\sqrt{2}},0)\ (-\frac{1}{\sqrt{2}},0)$ x intercepts

(5) [Marks: 8] Find the following limits by simplifying the expression first.

(a) $\displaystyle\lim_{t\to 1} \frac{1}{t-1}\left[\frac{1}{(t+3)^2} - \frac{1}{16}\right] = \lim_{t\to 1} \frac{1}{t-1}\left[\frac{16-(t+3)^2}{16(t+3)^2}\right]$

$\displaystyle = \lim_{t\to 1} \frac{1}{t-1}\left[\frac{16-t^2-6t-9}{16(t+3)^2}\right] = \lim_{t\to 1} \frac{1}{t-1}\left[\frac{-(t-1)(t+7)}{16(t+3)^2}\right]$

$\displaystyle = \lim_{t\to 1} -\frac{t+7}{16(t+3)^2} = -\frac{1+7}{16(1+3)^2} = \frac{-8}{16\cdot 16} = -\frac{1}{32}$

(b) $\lim_{u \to 5} \dfrac{\sqrt{u+4} - 3}{u-5} = \lim_{u \to 5} \dfrac{\sqrt{u+4} - 3}{u-5} \cdot \left(\dfrac{\sqrt{u+4} + 3}{\sqrt{u+4} + 3} \right)$

$$= \lim_{u \to 5} \frac{u+4-9}{u-5\left(\sqrt{u+4}+3\right)} = \lim_{u \to 5} \frac{u-5}{u-5\left(\sqrt{u+4}+3\right)}$$

$$= \lim_{u \to 5} \frac{1}{\sqrt{u+4}+3} = \frac{1}{\sqrt{5+4}+3} = \frac{1}{6}$$

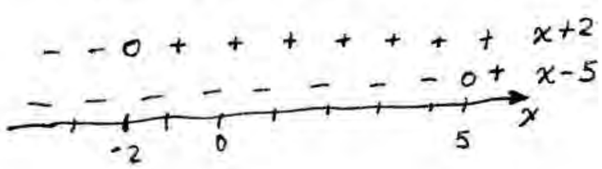
(6) [Marks:5] Find the domain of the following function.

$$f(x) = \frac{\sqrt{x^2 - 3x - 10}}{x^4 - 16}$$

require $x^2 - 3x - 10 \geq 0$ <u>and</u> $x^4 - 16 \neq 0$.

$x^2 - 3x - 10 \geq 0$

$\Leftrightarrow (x-5)(x+2) \geq 0$



$(-\infty, -2] \cup [5, \infty)$

$x^4 - 16 = (x^2-4)(x^2+4)$

$x^2 - 4 = (x+2)(x-2)$

so $x^4 - 16 = 0$ if $x = 2, -2$

exclude!

Answer is intersection:

$(-\infty, -2) \cup [5, \infty)$

(7) [Marks:5] Find the equation of the line that is perpendicular to the line $2x - 3y = 5$ and passes through the point $(-3, 4)$.

Slope of $2x-3y=5$ is $2/3$, so slope of line we are looking for is $-3/2$. Use point-slope formula;

$$-\frac{3}{2} = \frac{y-4}{x+3} \implies -3x-9 = 2y-8$$

or $2y+3x=-1$

or $y = -\frac{3}{2}x - \frac{1}{2}$

(8) [Marks: 15] Consider the quadratic function $f(x) = -3x^2 + 24x - 36$.

(a) Express the function in standard form and from this find the vertex and axis of symmetry.

Complete square; $-3x^2+24x-36$

$= -3(x^2-8x+12)$

$= -3\left((x-4)^2-4\right) = -3(x-4)^2+12$

4 marks

axis of symmetry; $x=4$

vertex: $(4, 12)$

(b) Find <u>all</u> intercepts.

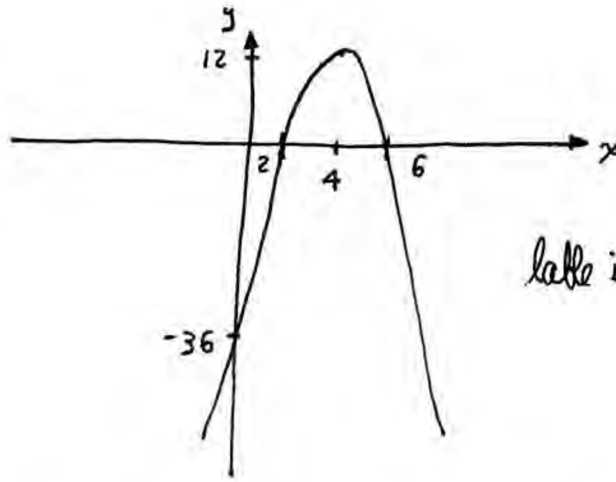
y intercept; $f(0) = -36$

x intercepts; $x_{1,2} = 4 \pm \frac{\sqrt{16}}{2} = 2, 6$

3 marks

(also see this by factoring: $-3x^2+24x-36 = -3(x^2-8x+12)$
$= -3(x-2)(x-6)$)

(c) Make a sketch of the graph.



label intercepts

2 marks

(d) Find the points of intersection between this parabola and the line $y = 3x - 3$.

At point (x, y) of intersection, have that

$$3x - 3 = -3x^2 + 24x - 36 \Rightarrow 3x^2 - 21x + 33 = 0$$

$$\text{or } x^2 - 7x + 11 = 0$$

3 marks

Use quadratic formula to solve;

$$x = \frac{7 \pm \sqrt{49 - 44}}{2} = \frac{7}{2} \pm \frac{\sqrt{5}}{2} \; ; \; x_1, x_2$$

The corresponding y values are $y_1 = 3x_1 - 3$, $y_2 = 3x_2 - 3$

(x_1, y_1), (x_2, y_2) are the two points of intersection.

(e) From this, sketch the region in the xy-plane of the points (x, y) that satisfy the inequalities

$$3x - 3 \leq y \leq -3x^2 + 24x - 36$$

3 marks

Note that the points satisfying $y \geq 3x - 3$ lie above the line $y = 3x - 3$,
(for example, $(0,0)$ satisfies this)
and the points satisfying $y \leq -3x^2 + 24x - 36$
lie below the parabola (for example, the point $(4, 0)$ satisfies this).
So we see that the points satisfying both inequalities lies between the two curves: