Math 100 Final 2008

(1) [Marks: 8] Solve the following inequalities or equations. For the inequalities, express your answer in interval notation.

(a) $-2 \le \frac{4-3x}{2+x} < 3$

$-2 \le \frac{4-3x}{2+x}$ mult $\cdot(2+x)$

$0 \le \frac{4-3x+4+2x}{2+x}$

$0 \le \frac{8-x}{2+x}$

$8-x$	+	+ + +	− − −	
$2+x$	−	+	+ +	+ +
	-2	0	8	

1

$\boxed{(-2, 8]}$

$\frac{4-3x}{2+x} < 3$

$\frac{4-3x-6-3x}{2+x} < 0$

$\frac{-2-6x}{2+x}$

$-2-6x$	+	+	− − − −
$2+x$	−	+	+ +
	-2	$-\frac{1}{3}$	

$(-\infty, -2) \cup (-\frac{1}{3}, \infty)$

intersection :

$(-\frac{1}{3}, 8]$

(b) $\left|\frac{3x-2}{2}\right| > 6-x$

$\frac{3x-2}{2} > 6+x$

$3x-2 > 12+2x$

$x > 14$

$\frac{3x-2}{2} < -6-x$

$3x-2 < -12-2x$

$5x < -10$

$x < -2$

$\boxed{\text{union} \quad (-\infty, -2) \cup (14, \infty)}$

(c) $|1-x^2| = 2$

$1-x^2 = 2$

$x^2 = -1$

No soln

$1-x^2 = -2$

$x^2 = 3$

$x = \pm\sqrt{3}$

(2) [Marks: 6] Find the following limits by simplifying the expression first.

(a) $\lim_{x\to 25} \frac{25-x}{5-\sqrt{x}} \cdot \frac{5+\sqrt{x}}{5+\sqrt{x}} = \frac{(25-x)(5+\sqrt{x})}{25-x} = 5+\sqrt{x} \to 5+\sqrt{25} = \boxed{10}$

1

(b) $\lim_{x\to -3} \frac{x^3+4x^2+3x}{x^2+2x-3} = \lim \frac{x(x+3)(x+1)}{(x-1)(x+3)} = \lim \frac{x(x+1)}{x-1} = \frac{(-3)(-3+1)}{-3-1}$

$= \frac{(-3)(-2)}{-4} = \frac{6}{-4} = \boxed{-\frac{3}{2}}$

0

(3) [Marks: 5] Determine a value of d so that the two parabolas $-(x-2)^2+d$ and $(x+1)^2-3$ intersect
(a) exactly once, (b) exactly twice

$-(x-2)^2+d = (x+1)^2-3$

$-(x^2-4x+4)+d = x^2+2x+1-3$

$2x^2-2x+(2-d) = 0$

discriminant: $b^2-4ac = 4-8(2-d)$

$= 4-16+8d$

$= -12+8d = 0$ one solution

$8d = 12$

$\boxed{d = \frac{12}{8} = \frac{3}{2}}$ one solution

$\boxed{d > \frac{3}{2}}$ two solutions

0

(4) [Marks: 7] For each of the following functions, prove either that $f(x)$ is not one-to-one or that $f(x)$ is one-to-one. If $f(x)$ is one-to-one, determine the inverse $f^{-1}(x)$, the domains and ranges of both f and f^{-1}, and verify that your formula for $f^{-1}(x)$ is indeed the inverse of $f(x)$.

(a) $y = \frac{|x+2|-3}{x^2+1}$

roots: $|x+2|-3 = 0$

$|x+2| = 3$

$x+2=3 \to x=1$

$x+2=-3 \to x=-5$

$\left.\begin{matrix} \\ \end{matrix}\right\} y(1)=0=y(5)$

So y has two roots $\to$ not 1-1

(b) $y = 2-\sqrt{x^3+1}$

$\underline{y(a)=y(b)}$: $2-\sqrt{a^3+1} = 2-\sqrt{b^3+1}$

$\to \sqrt{a^3+1} = \sqrt{b^3+1}$

$\to a^3+1 = b^3+1 \to a^3=b^3 \to \underline{\underline{a=b}}$ is 1-1

Find f^{-1}: $y=2-\sqrt{x^3+1}$

$y-2=-\sqrt{x^3+1}$

$2-y = \sqrt{\ }$

$(y-2)^2 = x^3+1$

$\to x = \sqrt[3]{(y-2)^2-1}$

$\boxed{f^{-1}(x) = \sqrt[3]{(x-2)^2-1}}$

$(2-x)^2$

domain f: $x^3+1 \geq 0$; $x \geq -1$

range f: $y \leq 2$

$\to$ range f^{-1}: $y \geq -1$

domain f^{-1}: $x \leq 2$

$(f \circ f^{-1})(x) = 2-\sqrt{(x-2)^2-1+1} = 2-\sqrt{(x-2)^2} = 2-(x-2)$

$(f^{-1} \circ f)(x) = \sqrt[3]{(\sqrt{x^3+1})^2-1} = \sqrt[3]{x^3} = x$

(5) [Marks: 6] Put the following equations into standard form and identify either the centre of the circle and its radius, or the axis of symmetry and vertex of the parabola.

(a) $y - 7 = \frac{1}{3}x^2 - 2x + 3$

$$\left.\begin{aligned} y &= \tfrac{1}{3}x^2 - 2x + 10 \\ &= \tfrac{1}{3}\left(x^2 - 6x + 30\right) \\ &= \tfrac{1}{3}\left((x-3)^2 + 21\right) \\ &= \tfrac{1}{3}(x-3)^2 + 7 \end{aligned}\right\}$$

$$\boxed{\begin{aligned} &x = 3 \\ &V = (3, 7) \end{aligned}}$$

(b) $10 + x^2 = 2 + 6y - 4x - y^2$

$$\left.\begin{aligned} &x^2 + y^2 + 4x - 6y + 8 = 0 \\ &(x+2)^2 - 4 + (y-3)^2 - 9 + 8 = 0 \end{aligned}\right\}$$

$$(x+2)^2 + (y-3)^2 = 5$$

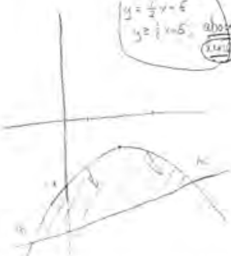
$$\boxed{\begin{aligned} &\text{centre } (-2, 3) \\ &\text{radius } \sqrt{5} \end{aligned}}$$

(6) [Marks: 4] Sketch the region in the plane (approximately; so no need to find intercepts or intersections exactly) that contains all the points (x, y) that satisfy the following inequalities:

$$\frac{1}{2}x - 6 \le y \le -2x^2 + 4x - 4$$

$y = \frac{1}{2}x - 6$
$y \ge \frac{1}{2}x - 6$, above (x,y)

$y = -2x^2 + 4x - 4$
$= -2(x^2 - 2x + 2)$
$= -2((x-1)^2 + 1) = -2(x-1)^2 - 2$
$y \le -2(x-1)^2 - 2$; below parabola.



(consistent with ✓)

(7) [Marks: 6] (a) Write out the formula for $f \circ g$ and $g \circ f$ where

$$f(x) = \frac{2(x-1)^2}{|x+1| + 1}, \qquad g(x) = \frac{1}{\sqrt{3x}} + 1$$

$f \circ g$: $2\left(\frac{1}{\sqrt{3x}}\right)^2 \Big/ \left|\frac{1}{\sqrt{3x}} + 2\right| + 1$

$\left(\frac{1}{\sqrt{3x}} + 1 - 1\right)^2, \left|\frac{1}{\sqrt{3x}} + 1 + 1\right|$

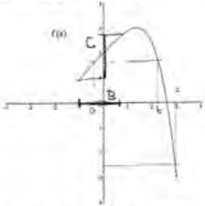
$g \circ f$: $1 \Big/ \sqrt{\frac{6(x-1)^2}{|x+1|+1}} + 1$

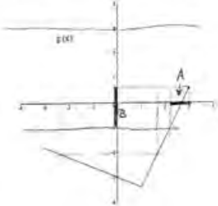
(b) Find two functions f and g, neither of which are the identity function $I(x) = x$ such that $h = f \circ g$ where

$$h(x) = \frac{\frac{1}{4}x + 2}{\sqrt{2x} + x}$$

$g = 2x$; $f = \dfrac{\frac{1}{8}x + 2}{\sqrt{x} + \frac{1}{2}x}$

(8) [Marks: 9] The graphs of two functions $f(x)$ and $g(x)$ are given below. The domain of $f(x)$ is $(-1,3]$. The domain of $g(x)$ is $(-3,3]$.





(a) Using the graphs, find all solutions to $g(f(x)) = -2$

require $f(x) = -2.5$ or 1.6

Have $f(a) = 1.6 = f(b)$

$f(c) = -2.5$

3 solutions: $a \approx -.2$

$b \approx 2.2$

$c \approx 2.9$

(b) Determine the domain and range of $f \circ g$. (Note that the compositions are different in parts (a) and (b).)

domain: need $g(x) \in \mathrm{dom}(f)$: $\boxed{(2.2, 3] = A}$; $g(A) = B = (-1, 0.6]$

range: is image $f(g(A)) = f(B) = \boxed{C = (1, 2.6]}$

(9) [Marks: 6] Determine the domains of the following functions. Express your answer in interval notation.

(a) $f(x) = \frac{x}{\sqrt{2x^2-4}} - 2\ln|x-2|$

A

1 $2x^2 - 4 > 0 \rightarrow 2x^2 > 4 \rightarrow x^2 > 2 \rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$|x-2| \neq 0$; $x \neq 2$

intersection : $A \setminus \{2\}$

1 $= (-\infty, -\sqrt{2}) \cup (\sqrt{2}, 2) \cup (2, \infty)$

(b) $g(x) = \left(-3\cos^{-1}(3-x)\right)\left(\tan \pi x\right)$

$\cos^{-1}$: 1 $-1 \leq 3 - x \leq 1$

$-4 \leq -x \leq -2$

$4 \geq x \geq 2$ $\leftarrow A = [2, 4]$

$\tan$: 1 $\pi x \neq \frac{2n+1}{2}\pi \rightarrow x \neq \frac{2n+1}{2} = B = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$

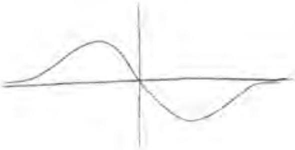
intersection : $[2, 4] \setminus \{\frac{5}{2}, \frac{7}{2}\}$

1 $= [2, \frac{5}{2}) \cup (\frac{5}{2}, \frac{7}{2}) \cup (\frac{7}{2}, 4]$

(10) [Marks 10] Sketch the graphs of the following rational functions. Begin by determining if the graph is symmetric (even/odd) and find any intercepts. If there are any asymptotes, be sure to determine how the graph approaches the asymptote.

(a) $R(x) = \frac{-2x}{x^4 + 3}$ Symmetry: ✓ odd Intercepts: only $(0,0)$

$R(-x) = -R(x)$



continued →

(b) $Q(x) = \frac{x^2 - 5x + 6}{4 - x}$ Symmetry: None

Intercepts: $Q(0) = \frac{6}{4} = \frac{3}{2} \leftarrow y$

$x^2 - 5x + 6 = (x-3)(x-2)$;

$x = 2, 3$

$$4-x \,\overline{\smash{)}\, x^2 - 5x + 6}$$ quotient $-x+1$

$x^2 - 4x$

$-x + 6$

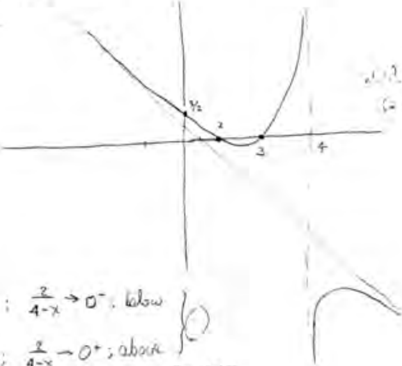
$-x + 4$

2

$Q(x) = -x + 1 + \frac{2}{4-x}$ slant: $-x+1$

vert asym: $x = 4$ $\lim_{x \to 4^+} Q = \frac{-}{-} \to \infty$ (1) $\lim_{x \to 4^-} Q = \frac{+}{+} \to +\infty$

$x^2 - 5x + 6$ @ $x = 4$: $16 - 20 + 6 \neq 0$



$x \to +\infty$: $\frac{2}{4-x} \to 0^-$; below

$x \to -\infty$: $\frac{2}{4-x} \to 0^+$; above

(11) (Marks 9) Consider the polynomial $p(x) = 3x(x+2)^3(x-3)^4$.

(a) Determine the shape of the graph of $p(x)$ near the roots (parts (i) - (iii) below). Make a sketch of the graph only near the root, and justify your answers.

(i) $x = 0$. $(x+2)>0$, $(x-3)>0$

$p(x) \approx kx$, $k>0$

(ii) $x = -2$:

$3x<0$, $(x-3)^4>0$

$p(x) \sim k(x+2)^3$, $k<0$

(iii) $x = 3$.

$3x>0$, $(x+2)^3>0$

$p(x) \sim k(x-3)^4$ $k>0$

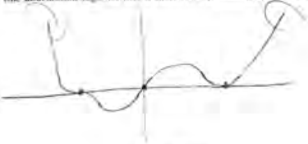
(b) What is the end behaviour of $p(x)$?

$p \approx 3x^8$

(c) What's the maximum number of points on the graph of $p(x)$ that have a horizontal tangent line?

$\deg p - 1 = 7$

(d) Put all this information together into a sketch of a possible graph of $p(x)$.



(12) [Marks: 7] (a) Write down the FORM of the partial fraction decomposition (but do NOT solve) for:

$$\frac{2x^3-7x+3}{(2x^2-x+1)(4-x)^2\boxed{(x^2+2x-8)}} \to (x-2)(x+4)$$

Irred.

$$= \frac{A}{4-x} + \frac{B}{(4-x)^2} + \frac{C}{(x-2)} + \frac{D}{(x+4)} + \frac{Ex+F}{2x^2-x+1}$$

(b) Find the partial fraction decomposition of

$$\frac{2x+1}{(x+2)(x^2-2x+3)}$$

Irred.

$$= \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+3}$$

$$\Rightarrow 2x+1 = A(x^2-2x+3) + (Bx+C)(x+2)$$

$$x=-2: \quad -3 = A(4+4+3) \to A = -3/11$$

$$x^2: \quad 0 = A+B \to B = -A = 3/11$$

$$x: \quad 2 = -2A+2B+C \to C = 2+2A-2B = 2-\frac{12}{11}$$

(13) [Marks: 4] A bar of length 4 metres is rotating about one end at an angular speed of 36° per minute. How fast, in <u>metres per minute</u>, is the other end of the bar (opposite the pivot point) moving? (Hint: Use the relation $s = \theta r$.) You can leave your answer in fractional form (no need to write it as a decimal).

arc length $s = \theta \cdot r$ radians × radius

$36°$ per min $= 360°/10\text{ min} = 2\pi\text{ rads}/10\text{ min}$

$= 0.2\pi\text{ rads}/\text{min}$

(other way)

$\rightarrow s = \frac{0.2\pi\text{ rads}}{\text{min}} \times 4\text{m} = 0.8\pi\text{ m}/\text{min}$

or, circumference $= 2\pi r = 8\pi$ m

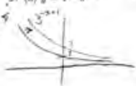
so, $8\pi\text{ m}/10\text{ min} = 0.8\pi\text{ m}/\text{min}$

/3

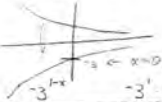
(14) [Marks 13] Sketch the graph of the following functions. State any x and y intercepts and asymptotes.

(a) $y = 4 - 3^{1-x}$

$3^{-x+1} = 3^{-(x-1)}$: 3^{-x} shifted right by 1



Then -3^{1-x}



Then shift up 4

$\leftarrow$ $\boxed{\text{asymptote } y=4}$

y int : 1

x int :

$4 - 3^{1-x} = 0$

$3^{1-x} = 4$

$1 - x = \log_3 4 \rightarrow x = 1 - \log_3 4 < 0$

(b) $y = \ln|x^2 - 1|$

domain $|x^2-1| \neq 0$

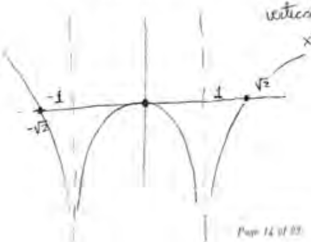
$x \neq \pm 1$

y int : $y(0) = \ln 1 = 0$

$x^2 < 1 \Rightarrow |x^2-1| < 1 \Rightarrow \ln < 0$

vertical asymptotes $x = \pm 1$

$x^2 = 2 \rightarrow \ln 1 = 0$ x-int $\pm\sqrt{2}$



continued →

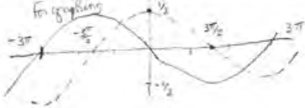
(c) $y = -\frac{1}{2}\sin\left(\frac{x}{3} - \frac{\pi}{2}\right) + 2$. State the period, amplitude and phase shift.

$\sin\left(\frac{1}{3}\left(x - \frac{3\pi}{2}\right)\right)$; $\sin \frac{1}{3}x$ shifted $\boxed{\frac{3\pi}{2}}$ right ← pmax $\frac{3\pi}{2}$

roots : $\frac{1}{3}x = n\pi$ → $\boxed{x = 3\pi n}$ $\boxed{\text{period } 6\pi}$

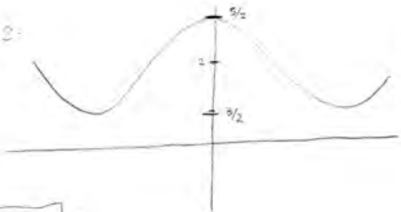
$-\frac{1}{2}\sin\frac{x}{3}$

For graphing



shift right $\frac{3\pi}{2}$ → - - - -

shift up 2 :



$\boxed{\text{amplitude} : \frac{1}{2}}$

(15) [Marks: 21] (a) Sketch on the unit circle (approximately), all angles θ such that $\csc\theta = -7/2$.

$\csc\theta = \frac{1}{\sin\theta}$; $\sin\theta = -\frac{2}{7}$

1

$\boxed{\theta_1 + 2n\pi, \quad \theta_2 + 2n\pi}$

(b) Determine the exact values of

(i) $\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$

(ii) $\tan^{-1}(-\sqrt{3}) = -\pi/3$ (not $5\pi/3$!)

(iii) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 3\pi/4$

3

(c) If $\sec x = -5$, $\pi/2 < x < \pi$, find the values of the remaining five trigonometric functions (at the same angle x).

$\sec x = -5 \rightarrow \boxed{\cos x = -\frac{1}{5}}$

4

$\rightarrow \sin x = +\sqrt{1 - \frac{1}{25}} = \boxed{\sqrt{24/25}}$

$\rightarrow \tan x = \frac{\sin x}{\cos x} = \boxed{\frac{\sqrt{24/25}}{-1/5}} = -5\sqrt{24/25}$

$\cot x = \frac{1}{\tan x} = \frac{-1/5}{\sqrt{24/25}}$

$\csc x = \frac{1}{\sin x} = \sqrt{25/24}$

continued →

(d) Find the **exact** value of $\sec\left(\frac{7\pi}{12}\right)$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\frac{7\pi}{6} = -\frac{1}{2}$

$\frac{x}{2} = \frac{7\pi}{12} \rightarrow x = \frac{7\pi}{6}$;

$\sec\frac{7\pi}{12} = \frac{1}{\cos(\frac{7\pi}{12})}$ $\cos\frac{7\pi}{12} = \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$= \pm\sqrt{\frac{1-\sqrt{3}/2}{2}}$; $-$ root $\rightarrow$ $\boxed{\sec\left(\frac{7\pi}{12}\right) = -\sqrt{\frac{2}{1-\sqrt{3}/2}}}$

may do it another way ($\cos\ \theta = \theta_1$)

must use reference angle

(e) Evaluate $\cos^{-1}(\cos(3\pi/2)) = \theta$; $\cos\theta = \cos(3\pi/2) = 0$

$\cos^{-1}(0) \in [0, \pi]$

so $\boxed{\theta = \pi/2}$

(f) Sketch (approximately) on the unit circle the angle θ such that $\theta = \arctan 3$. Then evaluate $\sin(\arctan 3)$

$\tan\theta = 3 = \frac{\sin\theta}{\cos\theta} \rightarrow \sin\theta = 3\cos\theta$

wrong $\rightarrow$

or $\cos\theta = \frac{1}{3}\sin\theta$

$\frac{1}{3}\sin\theta = \sqrt{1-\sin^2\theta}$

$\frac{1}{9}\sin^2\theta = 1 - \sin^2\theta$

$\frac{10}{9}\sin^2\theta = 1 \rightarrow \boxed{\sin\theta = \sqrt{\frac{9}{10}}}$

$3\cos\theta = \sin\theta = \sqrt{1-\cos^2\theta}$

$\rightarrow 9\cos^2\theta = 1 - \cos^2\theta$

$10\cos^2\theta = 1 \rightarrow \cos\theta = \frac{1}{\sqrt{10}} \rightarrow \boxed{\sin\theta = \sqrt{\frac{9}{10}}}$

continued $\rightarrow$

(g) Find an algebraic expression in x for $\cot(\underbrace{\cos^{-1} x}_{\theta})$.

$\cos\theta = x \rightarrow \sin\theta = \sqrt{1-x^2}$

$\theta \in [0,\pi]$ sin + root ✓

$$\rightarrow \boxed{\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{\sqrt{1-x^2}}}$$

(16) [Marks: 4] (a) If $z = 5^{-x}$, find 25^x in terms of z.

$$\frac{1}{z} = 5^x, \qquad 25^x = (5^2)^x = (5^x)^2 = \boxed{\left(\frac{1}{z}\right)^2 = \frac{1}{z^2} = z^{-2}}$$

(b) Find the exact value of $\left(\frac{1}{3}\right)^{\log_3 2}$

$$= (3^{-1})^{\log_3 2} = 3^{-\log_3 2} = \left(3^{\log_3 2}\right)^{-1} = \boxed{2^{-1} = \frac{1}{2}}$$

(17) [Marks: 15] Solve the following equations. Make sure you find <u>all</u> solutions. You do not need to simplify your answer (answers can be left as algebraic expressions, and if any angles you find are not reference angles, just label them as x_1, x_2 etc.); your answer should be written as $x = \cdots$

(a) $5\cos^2 x + 3\cos x = 0$ $\qquad a = \cos x$

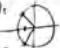
$5a^2 + 3a = 0$

$a(5a+3) = 0$; $\qquad a = 0$; $\quad \cos x = 0 \rightarrow x = \left(\frac{2n+1}{2}\right)\pi$

$5a+3 = 0 \rightarrow \boxed{a = -3/5}$

$\rightarrow \cos x = -3/5$; $\quad x = \theta_1 + 2n\pi$
$\qquad \theta_2 + 2n\pi$

θ_1



θ_2

all solutions: $x = \left(\frac{2n+1}{2}\right)\pi$, $\theta_1 + 2n\pi$,
$\theta_2 + 2n\pi$

(b) $2\cos 2x + \cos 4x = 0$ $\qquad \cos 4x = \cos^2 2x - \sin^2 2x = 2\cos^2 2x - 1$

$(1 - \cos^2 2x)$

$2\cos 2x + 2\cos^2 2x - 1 = 0$

$2a^2 + 2a - 1 = 0$

$\rightarrow a = \frac{-2-\sqrt{12}}{4} < -1$

or $\frac{-2+\sqrt{12}}{4}$ ✓

so $\cos 2x = \frac{-2+\sqrt{12}}{4} \sim \frac{1}{4}$

θ_1

θ_2

$2x = \theta_1 + 2n\pi$, $\theta_2 + 2n\pi$

$\rightarrow \boxed{x = \frac{\theta_1}{2} + n\pi}$ $\quad \frac{\theta_2}{2} + n\pi$

continued →

(c) $\log_5(x+1) + \log_5(x-1) - 2\log_5 x = -2$

$$\log_5 \left[\frac{(x+1)(x-1)}{x^2}\right] = -2$$

$$\rightarrow \frac{x^2-1}{x^2} = \frac{1}{25}$$

$$x^2 - 1 = \frac{1}{25}x^2$$

$$\frac{24}{25}x^2 = 1 \qquad x = \pm\frac{5}{\sqrt{24}}$$

(d) $\log_5 \frac{1}{\sqrt{x^2-3}} + 2 = 4$

$$-\frac{1}{2}\log_5 (x^2-3) = 2$$

$$\log_5 (\ ) = -4$$

$$x^2 - 3 = \frac{1}{5^4}$$

$$x^2 = 3 + \frac{1}{5^4}$$

$$x = \pm\sqrt{3 + \frac{1}{5^4}}$$

continued →

(b) $3^{(2x-1)} = 12$

$$2x-1 = \log_3 12$$

$$\rightarrow \; x = \frac{1}{2}\left(\log_3 12 + 1\right)$$

(18) [Marks: 4] (a) Let $x = 5^{\pi}$. Find two integers a and b, with $0 < a$, that lie above and below x (i.e., such that $0 < a < x < b$). Justify your answer.

$3 < \pi < 4$ so $5^3 < 5^{\pi} < 5^4$

$5^3 = 125$, $5^4 = 625$

(b) Let $y = \log_{\sqrt{3}} 20$. Find two consecutive integers a and $a+1$ that lie above and below y (i.e., find integer a such that $a < y < a+1$). Justify your answer.

$(\sqrt{3})^2 = 3, \quad (\sqrt{3})^4 = 9 \quad (\sqrt{3})^6 = 27$

$(\sqrt{3})^5 = \sqrt{3}\cdot 9 \;;\; \sqrt{3} < 2$ so $(\sqrt{3})^5 < 2\cdot 9 = 18 < 20$

$\rightarrow 18 < (\sqrt{3})^y = 20 < 27$

$\rightarrow \boxed{5 < y < 6}$

(19) [Marks: 4] The population of a country today is 14 million. 100 years ago the population was 10 million. Assuming the exponential model $P(t) = P_0 e^{kt}$ for the population, when will the population hit 25 million?

Find k:

$$P(-100) = 14\, e^{-k\cdot 100} = 10$$

$$e^{-100k} = \frac{5}{7} \rightarrow k = -\frac{\ln \frac{5}{7}}{100} = \frac{\ln \frac{7}{5}}{100} > 0$$

Find T such that

$$P(T) = 25 ; \quad 1$$

$$25 = 14\, e^{+k\cdot T} \qquad k =$$

$$\frac{25}{14} = e^{+kT}$$

$$T = \frac{\ln\left(\frac{25}{14}\right)}{k} .$$