MATH 100 - D200

Final Examination, December 2008

Instructor: R. Pyke   Time alloted: 180 minutes

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1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. NO CALCULATORS ARE ALLOWED. All numerical answers do not need to be simplified.
5. A **formula sheet** is attached at the end.
6. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

Question	Score	Max
1		8
2		4
3		4
4		7
5		6
6		4
7		6
8		9
9		6
10		10

Question	Score	Max
11		9
12		7
13		4
14		13
15		21
16		4
17		15
18		4
19		4
Total		145

(1) [Marks: 8] Solve the following inequalities or equations. For the inequalities, express your answer in interval notation.

(a) $-2 \le \dfrac{4-3x}{2+x} < 3$

(b) $\left|\dfrac{3x-2}{2}\right| > 6+x$

(c) $|1-x^2| = 2$

(2) [Marks: 6] Find the following limits by simplifying the expression first.

(a) $\displaystyle\lim_{x \to 25} \frac{25 - x}{5 - \sqrt{x}}$

(b) $\displaystyle\lim_{x \to -3} \frac{x^3 + 4x^2 + 3x}{x^2 + 2x - 3}$

(3) [Marks: 4] Determine a value of d so that the two parabolas $-(x-2)^2 + d$ and $(x+1)^2 - 3$ intersect **(a)** exactly once, **(b)** exactly twice.

(4) [Marks: 7] For each of the following functions, prove either that $f(x)$ is not one-to-one or that $f(x)$ is one-to-one. If $f(x)$ is one-to-one, determine the inverse $f^{-1}(x)$, the domains and ranges of both f and f^{-1}, and verify that your formula for $f^{-1}(x)$ is indeed the inverse of $f(x)$.

(a) $y = \dfrac{|x+2|-3}{x^2+1}$

(b) $y = 2 - \sqrt{x^3+1}$

(5) [Marks: 6] Put the following equations into standard form and identify either the centre of the circle and it radius, or the axis of symmetry and vertex of the parabola.

(a) $y - 7 = \frac{1}{3}x^2 - 2x + 3$

(b) $10 + x^2 = 2 + 6y - 4x - y^2$

(6) [Marks: 4] Sketch the region in the plane (approximately; so no need to find intercepts or intersections exactly) that contains all the points (x, y) that satisfy the following inequalities;

$$\frac{1}{2}x - 6 \quad \leq \quad y \quad \leq -2x^2 + 4x - 4$$

(7) [Marks: 6] (a) Write out the formula for $f \circ g$ and $g \circ f$ where

$$f(x) = \frac{2(x-1)^2}{|x+1| \; + 1}, \qquad g(x) = \frac{1}{\sqrt{3x}} + 1$$

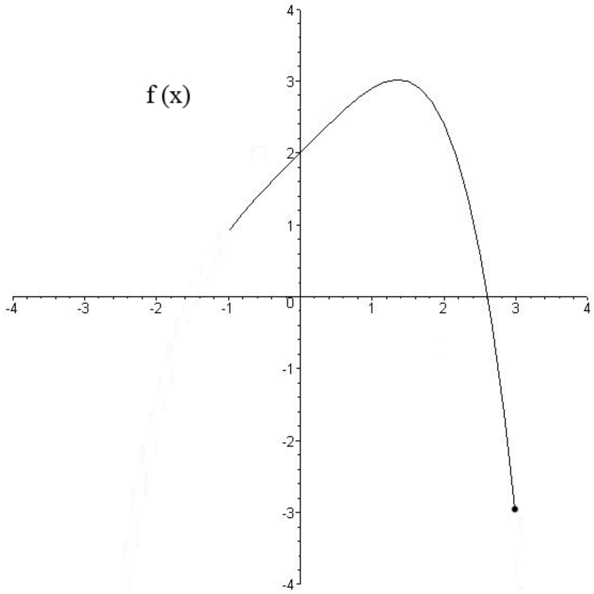
$f \circ g$:

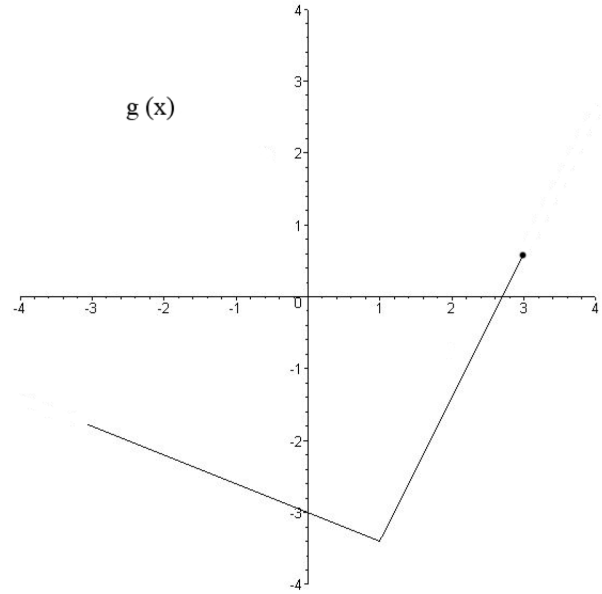
$g \circ f$:

(b) Find two functions f and g, neither of which are the identity function $I(x) = x$, such that $h = f \circ g$ where

$$h(x) = \frac{\frac{1}{4}x + 2}{\sqrt{2x} \; + x}$$

(8) [Marks: 9] The graphs of two functions $f(x)$ and $g(x)$ are given below. The domain of $f(x)$ is $(-1, 3]$. The domain of $g(x)$ is $(-3, 3]$.





(a) Using the graphs, find <u>all</u> solutions to $g(f(x)) = -2$

(b) Determine the <u>domain</u> and <u>range</u> of $f \circ g$. (Note that the compositions are different in parts (a) and (b).)

(9) [Marks: 6] Determine the domains of the following functions. Express your answer in interval notation.

(a) $f(x) = \dfrac{x}{\sqrt{2x^2 - 4}} - 2\ln|x-2|$

(b) $g(x) = \left(-3\cos^{-1}(3-x)\right)\left(\tan \pi x\right)$

(10) [Marks: 10] Sketch the graphs of the following rational functions. Begin by determining if the graph is symmetric (even/odd) and find any intercepts. If there are any asymptotes, be sure to determine how the graph approaches the asymptote.

(a) $R(x) = \dfrac{-2x}{x^4 + 3}$ Symmetry: Intercepts:

continued →

(b) $Q(x) = \dfrac{x^2 - 5x + 6}{4 - x}$ Symmetry: Intercepts:

(11) [Marks: 9] Consider the polynomial $p(x) = 3x(x+2)^3(x-3)^4$.

(a) Determine the shape of the graph of $p(x)$ near the roots (parts (i) - (iii) below). Make a sketch of the graph only near the root, and justify your answers.

(i) $x = 0$;

(ii) $x = -2$;

(iii) $x = 3$;

(b) What is the end behaviour of $p(x)$?

(c) What's the maximum number of points on the graph of $p(x)$ that have a horizontal tangent line?

(d) Put all this information together into a sketch of a possible graph of $p(x)$.

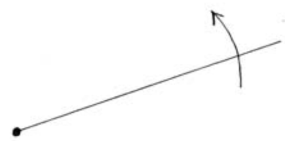
(12) [Marks: 7] (a) Write down the FORM of the partial fraction decomposition (but do NOT solve) for;

$$\frac{2x^3 - 7x + 3}{(2x^2 - x + 1)(4 - x)^2(x^2 + 2x - 8)}$$

(b) Find the partial fraction decomposition of

$$\frac{2x + 1}{(x + 2)(x^2 - 2x + 3)}$$

(13) [Marks: 4] A bar of length 4 metres is rotating about one end at an angular speed of 36^o per minute. How fast, in metres per minute, is the other end of the bar (opposite the pivot point) moving? (Hint: Use the relation $s = \theta r$.) You can leave your answer in fractional form (no need to write it as a decimal).



(14) [Marks: 13] Sketch the graph of the following functions. State any x and y intercepts and asymptotes.

(a) $y = 4 - 3^{1-x}$

(b) $y = \ln|x^2 - 1|$

continued →

(c) $y = -\dfrac{1}{2}\sin\left(\dfrac{x}{3} - \dfrac{\pi}{2}\right) + 2.$ State the period, amplitude and phase shift.

(15) [Marks: 21] (a) Sketch on the unit circle (approximately), all angles θ such that $\csc\theta = -7/2$.

(b) Determine the exact values of

(i) $\sin^{-1}\left(\frac{1}{2}\right)$

(ii) $\tan^{-1}(-\sqrt{3})$

(iii) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(c) If $\sec x = -5$, $\pi/2 < x < \pi$, find the values of the remaining five trigonometric functions (at the same angle x).

continued →

(d) Find the exact value of $\sec\left(\dfrac{7\pi}{12}\right)$

(e) Evaluate $\cos^{-1}(\cos(3\pi/2))$

(f) Sketch (approximately) on the unit circle the angle θ such that $\theta = \arctan 3$. Then evaluate $\sin(\arctan 3)$

continued →

(g) Find an algebraic expression in x for $\cot(\cos^{-1} x)$.

(16) [Marks: 4] (a) If $z = 5^{-x}$, find 25^x in terms of z.

(b) Find the exact value of $\left(\frac{1}{3}\right)^{\log_3 2}$

(17) [Marks: 15] Solve the following equations. Make sure you find <u>all</u> solutions. You do not need to simplify your answer (answers can be left as algebraic expressions, and if any angles you find are not reference angles, just label them as x_1, x_2 etc.); your answer should be written as $x = \cdots$.

(a) $5\cos^2 x + 3\cos x = 0$

(b) $2\cos 2x + \cos 4x = 0$

continued →

(c) $\log_5(x+1) + \log_5(x-1) - 2\log_5 x = -2$

(d) $\log_5 \dfrac{1}{\sqrt{x^2-3}} \; + 2 \; = \; 4$

continued $\rightarrow$

(e) $3^{(2x-1)} = 12$

(18) [Marks: 4] (a) Let $x = 5^{\pi}$. Find two integers a and b, with $0 < a$, that lie above and below x (i.e., such that $0 < a < x < b$). Justify your answer.

(b) Let $y = \log_{\sqrt{3}} 20$. Find two <u>consecutive</u> integers a and $a+1$ that lie above and below y (i.e., find integer a such that $a < y < a+1$). Justify your answer.

(19) [Marks: 4] The population of a country today is 14 million. 100 years ago the population was 10 million. Assuming the exponential model $P(t) = P_o e^{kt}$ for the population, when will the population hit 25 million?

Formulae

$$\csc x = \frac{1}{\sin x}, \qquad \sec x = \frac{1}{\cos x}, \qquad \cot x = \frac{1}{\tan x}$$

$$\begin{aligned}
\sin 2x &= 2\,\sin x\,\cos x, \\
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 1 - 2\sin^2 x \\
&= 2\cos^2 x - 1, \\
\tan 2x &= \frac{2\tan x}{1 - \tan^2 x}
\end{aligned}$$

$$\begin{aligned}
\sin\frac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos\frac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}} \\
\tan\frac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}
\end{aligned}$$

$$\begin{aligned}
\sin(u \pm v) &= \sin u\,\cos v \pm \cos u\,\sin v, \\
\cos(u \pm v) &= \cos u\,\cos c \mp \sin u\,\sin v, \\
\tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u\,\tan v}
\end{aligned}$$