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RESEARCH ARTICLE

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All group-based latin squares possess near transversals

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Abstract

In a latin square of order n, a near transversal is a collection of $n-1$ cells which intersects each row, column, and symbol class at most once. A longstanding conjecture of Brualdi, Ryser, and Stein asserts that every latin square possesses a near transversal. We show that this conjecture is true for every latin square that is main class equivalent to the Cayley table of a finite group.

KEYWORDS

Brualdi's conjecture, Cayley table, complete mapping, latin square, partial transversal

1 | INTRODUCTION AND MAIN THEOREMS

A *latin square* of order n is an $n \times n$ array in which each row and column is a permutation of some set of n symbols. We refer to the set of cells containing any fixed symbol as a *symbol class*. Let $L = [L_{i,j}]$ be a latin square of order n. A *partial transversal* of L is a collection of cells which intersects each row, column, and symbol class at most once. A *transversal* is a partial transversal of size n and a *near transversal* is a partial transversal of size $n-1$. Although it is straightforward to find latin squares possessing no transversals (see [22, p. 405]), there is no known example of a latin square which does not possess a near transversal.

Conjecture 1. *Every latin square possesses a near transversal.*

First discussed in the literature roughly 50 years ago, Conjecture 1 has been variously attributed to Brualdi, Ryser, and Stein (see [22, Section 5]). The strongest general lower bound to date is due to Hatami and Shor [15], who showed that every latin square possesses a partial transversal of size $n - O(\log^2(n))$. There have also been numerous attempts to establish this conjecture as a special case of some stronger statement, including work in terms of hypergraph matchings [1, 2, 10], covering radii of sets of permutations [6, 16, 19], and colorings of strongly regular graphs [4, 7, 11]. The present paper approaches Conjecture 1 from the opposite direction by proving its most widely discussed special case (see [18, p. 335]).

It is well-known that latin squares are equivalent to multiplication tables of finite quasigroups [18, Section 1.1]. When a latin square L corresponds to the multiplication table of an associative quasigroup (ie, a group) G then we say that L is *group-based*. In this case we refer to L as the *Cayley table* of G and write $L = L(G)$ to indicate the relationship between L and G.

A theorem of Hall [13] implies that, for every Abelian group G, the group-based latin square $L(G)$ possesses a near transversal. It has also been known since the 1950s that $L(G)$ possesses a transversal (from which we obtain a near transversal by removing any cell) whenever G has odd order [5], or G is solvable and has noncyclic Sylow 2-subgroups [14]. More recently, Evans and Wilcox [8, 23] characterized the groups G for which $L(G)$ possesses a transversal. Building on this characterization, we establish the restriction of Conjecture 1 to group-based latin squares.

Theorem 2. *Every group-based latin square possesses a near transversal.*

It is worth noting that Theorem 2 can be trivially extended to a slightly wider class of latin squares. The existence of a near transversal is not affected by relabelling the rows, columns, or symbols of L, nor is it affected by permuting the roles played by rows, columns, and symbols. Thus, every latin square which is main class equivalent (see [18, Section 1.4]) to a group-based latin square possesses a near transversal by Theorem 2.

We prove Theorem 2 using a graph-theoretic technical lemma; we refer the reader to [12] for background information and any undefined terms related to graphs. Letting $\mathcal{I}$ denote the set indexing the rows and columns of the latin square L, the *latin square graph* $\Gamma(L)$ is defined on the vertex set $\{(r, c) : r, c \in \mathcal{I}\}$ with $(r, c) \sim (s, d)$ if and only if one of $r = s$, $c = d$, or $L_{r,c} = L_{s,d}$ holds. Note that there is a natural tripartition of $\Gamma(L)$'s edges into, respectively, *row edges*, *column edges*, and *symbol edges*. Moreover, there is a bijective correspondence between near transversals of L and independent sets of size $n-1$ in $\Gamma(L)$.

Given graphs $\Gamma = (V, E)$ and $\Gamma = (V', E')$, the *disjoint union* of Γ and Γ' is $\Gamma + \Gamma' := (V \sqcup V', E \sqcup E')$. For a positive integer k, we write $k\Gamma$ for the disjoint union of k copies of Γ. Given a set $W \subseteq V$, the *induced subgraph* of Γ with respect to W is $\Gamma[W] := (W, \{e \in E : e \subseteq W\})$. The *Möbius ladder* of order $2n$, denoted as M_n, is the cubic graph formed from a cycle of length $2n$—referred to as the *rim* of M_n—by adding n edges, one joining each pair of vertices at distance n in the initial cycle. The *prism graph* of order $2n$, denoted as Y_n, is the Cartesian product of a cycle of length n and an edge; in other words, Y_n is obtained from two copies of the n-cycle by adding edges between corresponding vertices.

Lemma 3. *Let L be a group-based latin square of even order n, let k be the greatest power of 2 dividing n, and let $l := n/k$. If L does not possess a transversal, then there is a positive integer m dividing l such that $\Gamma(L)$ has an induced subgraph isomorphic to*

$$\Lambda_{n,m} := M_{km} + \left(\frac{l-m}{2}\right) Y_{2k}.$$

Section 2 will be devoted to proving Lemma 3, while Section 3 mentions two possible extensions of the present work. We conclude this section by proving Theorem 2 assuming Lemma 3.

Proof of Theorem 2. Let L be a group-based latin square of order n. We may assume L does not possess a transversal. As first shown in [5], this implies n is even. We may therefore, apply Lemma 3 to find an induced copy of $\Lambda_{n,m}$ in $\Gamma(L)$. Because the $(2k(l-m))$-vertex graph $\binom{\frac{l-m}{2}}{}Y_{2k}$ is bipartite, it contains an independent set of size at least $k(l-m)$. Moreover, one can find an independent set of size $km-1$ in M_{km} by greedily selecting vertices in cyclic order around its rim. Thus $\Lambda_{n,m}$ contains an independent set of size $(l-m)k+km-1=n-1$ which corresponds to a near transversal of L. □

2 | PROOF OF LEMMA 3

Let G be a group of order n with identity element e and let $Syl_2(G)$ denote the isomorphism class of G's Sylow 2-subgroups. A bijection $\sigma: G \to G$ is called a *complete mapping* of G if the set $\{L(G)_{g,\sigma(g)}: g \in G\}$ is a transversal of $L(G)$. Using the classification of finite simple groups, partial results of Hall and Paige, and an as yet unpublished computational result due to Bray, Evans, and Wilcox recently characterized the groups possessing complete mappings.

Theorem 4 (Evans [8], Hall and Paige [14], and Wilcox [23]). *A finite group G possesses a complete mapping if and only if $Syl_2(G)$ is either trivial or noncyclic.*

It is not hard to check that if H is a group of odd order, then the identity map is a complete mapping. Another nicely structured complete mapping for groups of odd order was found by Beals, Gallian, Headley, and Jungreis.

Lemma 5 (Beals et al. [3]). *For every group H of odd order m, there exists an ordering $H=\{h_0, h_1, \ldots, h_{m-1}\}$ such that, taking indices modulo m, both $h_i \mapsto h_{i+1}$ and $h_i \mapsto h_i$ are complete mappings.*

Given two subsets $X_1, X_2 \subseteq G$ the *product set* of X_1 by X_2 is

$$X_1X_2 := \{x_1x_2 : x_1 \in X_1, x_2 \in X_2\}.$$

We write Xy for the product set $X\{y\}$. Let K be a subgroup of G and let H be a normal subgroup of G. We say that G is *the semidirect product* of K and H, written $G=K \ltimes H$, if $K \cap H=\{e\}$, $KH=G$, and $|G|=|K|\,|H|$. The following was noted in [14] as following from a result of Burnside.

Lemma 6 (Burnside; Hall and Paige [14]). *Let G be a finite group and let K be a Sylow 2-subgroup of G. If K is cyclic and nontrivial, then there is a normal subgroup of odd order $H \lhd G$ such that*

$$G = K \ltimes H.$$

To simplify notation we set $[n] := \{0, 1, \ldots, n-1\}$ for every positive integer n.

Proof of Lemma 3. Let L be a latin square based on a group G of order $n=kl$, where $k \geq 2$ is a power of 2 and l is odd. Moreover, suppose L does not possess a transversal.

Theorem 4 tells us that $Syl_2(G) = \mathbb{Z}_k$. It then follows from Lemma 6 that G contains a normal subgroup H of order l and an element b of order k such that

$$G = \langle b \rangle \ltimes H.$$

Let $a := b^{k/2}$. As $H \lhd G$ and a has order 2, H has an automorphism

$$\alpha : h \mapsto aha.$$

Let

$$H^* := \{h \in H : \alpha(h) = h\}$$

and observe that H^* is a subgroup of H. Let $m := |H^*|$. As m divides l and l is odd, m is odd. By Lemma 5, there is an ordering $H^* = \{h_0, h_1, ..., h_{m-1}\}$ for which the map $h_i \mapsto h_{i+1}$ is a complete mapping. Here and throughout the rest of this proof, indices are taken modulo m.

Let $\Gamma := \Gamma(L)$. Toward defining a set $W \subseteq V(\Gamma)$ which induces $\Lambda_{n,m}$, let

$$\begin{aligned} T_1 &:= \{(b^i h_i, h_i b^i) \ : i \in [km]\}, \\ T_2 &:= \{(b^i h_i, h_{i+1} b^{i+1}) \ : i \in [km]\}, \quad \text{and} \\ T &:= T_1 \cup T_2. \end{aligned}$$

Furthermore, let $F := H \backslash H^*$ and let

$$\begin{aligned} U_1 &:= \{(b^i f, f b^i) : f \in F, i \in [k]\}, \\ U_2 &:= \{(b^i f, f b^{i+1}) : f \in F, i \in [k]\}, \quad \text{and} \\ U &:= U_1 \cup U_2. \end{aligned}$$

Finally, let

$$W := T \cup U.$$

See Figure 1 for an example of this construction. We show the induced subgraph $\Gamma[W] \cong \Lambda_{n,m}$ via the following three "14" claims.

Claim 1. $T \cap U = \varnothing$ and there is no edge between T and U.

As $G = \langle b \rangle \ltimes H$, every element of G has a unique representation of the form $g = b^i h$ for $i \in [k]$ and $h \in H$. Therefore, the definition of F implies

$$\langle b \rangle H^* \cap \langle b \rangle F = \varnothing. \tag{1}$$

But for every $(t, s) \in T$ and every $(u, v) \in U$ we have $t \in \langle b \rangle H^*$ and $u \in \langle b \rangle F$. Thus $T \cap U = \varnothing$ and there are no row edges between T and U.

$\mathbf{1}$	$\mathbf{bc}$	c^2	b	c	bc^2	d	d^2	cd	cd^2	c^2d	c^2d^2	bd^2	bd	bcd^2	bcd	bc^2d^2	bc^2d
bc	$\mathbf{c^2}$	$\mathbf{b}$	c	bc^2	1	bcd	bcd^2	bc^2d	bc^2d^2	bd	bd^2	cd^2	cd	c^2d^2	c^2d	d^2	d
c^2	b	$\mathbf{c}$	$\mathbf{bc^2}$	1	bc	c^2d	c^2d^2	d	d^2	cd	cd^2	bc^2d^2	bc^2d	bd^2	bd	bcd^2	bcd
b	c	bc^2	$\mathbf{1}$	$\mathbf{bc}$	c^2	bd	bd^2	bcd	bcd^2	bc^2d	bc^2d^2	d^2	d	cd^2	cd	c^2d^2	c^2d
c	bc^2	1	bc	$\mathbf{c^2}$	$\mathbf{b}$	cd	cd^2	c^2d	c^2d^2	d	d^2	bcd^2	bcd	bc^2d^2	bc^2d	bd^2	bd
$\mathbf{bc^2}$	1	bc	c^2	b	$\mathbf{c}$	bc^2d	bc^2d^2	bd	bd^2	bcd	bcd^2	c^2d^2	c^2d	d^2	d	cd^2	cd
d	bcd^2	c^2d	bd^2	cd	bc^2d^2	$\mathbf{d^2}$	1	cd^2	c	c^2d^2	c^2	$\mathbf{bd}$	b	bcd	bc	bc^2d	bc^2
d^2	bcd	c^2d^2	bd	cd^2	bc^2d	1	$\mathbf{d}$	c	cd	c^2	c^2d	b	$\mathbf{bd^2}$	bc	bcd^2	bc^2	bc^2d^2
cd	bc^2d^2	d	bcd^2	c^2d	bd^2	cd^2	c	$\mathbf{c^2d^2}$	c^2	d^2	1	bcd	bc	$\mathbf{bc^2d}$	bc^2	bd	b
cd^2	bc^2d	d^2	bcd	c^2d^2	bd	c	cd	c^2	$\mathbf{c^2d}$	1	d	bc	bcd^2	bc^2	$\mathbf{bc^2d^2}$	b	bd^2
c^2d	bd^2	cd	bc^2d^2	d	bcd^2	c^2d^2	c^2	d^2	1	$\mathbf{cd^2}$	c	bc^2d	bc^2	bd	b	$\mathbf{bcd}$	bc
c^2d^2	bd	cd^2	bc^2d	d^2	bcd	c^2	c^2d	1	d	c	$\mathbf{cd}$	bc^2	bc^2d^2	b	bd^2	bc	$\mathbf{bcd^2}$
bd	cd^2	bc^2d	d^2	bcd	c^2d^2	$\mathbf{bd^2}$	b	bcd^2	bc	bc^2d^2	bc^2	$\mathbf{d}$	1	cd	c	c^2d	c^2
bd^2	cd	bc^2d^2	d	bcd^2	c^2d	b	$\mathbf{bd}$	bc	bcd	bc^2	bc^2d	1	$\mathbf{d^2}$	c	cd^2	c^2	c^2d^2
bcd	c^2d^2	bd	cd^2	bc^2d	d^2	bcd^2	bc	$\mathbf{bc^2d^2}$	bc^2	bd^2	b	cd	c	$\mathbf{c^2d}$	c^2	d	1
bcd^2	c^2d	bd^2	cd	bc^2d^2	d	bc	bcd	bc^2	$\mathbf{bc^2d}$	b	bd	c	cd^2	c^2	$\mathbf{c^2d^2}$	1	d^2
bc^2d	d^2	bcd	c^2d^2	bd	cd^2	bc^2d^2	bc^2	bd^2	b	$\mathbf{bcd^2}$	bc	c^2d	c^2	d	1	$\mathbf{cd}$	c
bc^2d^2	d	bcd^2	c^2d	bd^2	cd	bc^2	bc^2d	b	bd	bc	$\mathbf{bcd}$	c^2	c^2d^2	1	d^2	c	$\mathbf{cd^2}$

FIGURE 1 Cayley table of $S_3 \times \mathbb{Z}_3 = \langle b, c, d \mid b^2 = c^3 = d^3 = 1, bc = cb, bd = d^2b \rangle$ with $\underline{\mathbf{T}}$ and $\mathbf{U}$ highlighted. Here $k = 2$, $H = \langle c, d \rangle$, and $H^* = \langle c \rangle$, with H^* ordered by $h_i = c^i$. The first six rows and columns are indexed by $\langle b \rangle H^*$ and the main diagonal is $T_1 \cup U_1$ [Color figure can be viewed at wileyonlinelibrary.com]

As $H \triangleleft G$ we have $bHb^{-1} = H$. Moreover, as $a = b^{k/2}$, for every $h \in H^*$ we have $\alpha(bhb^{-1}) = bhb^{-1}$, so $bhb^{-1} \in H^*$. Thus

$$bH^*b^{-1} = H^*. \tag{2}$$

It then follows from the definition of F that

$$bFb^{-1} = F \tag{3}$$

and, as the identity map is a complete mapping of both H and H^*,

$$f \mapsto f^2 \text{ is a permutation of } F \tag{4}$$

Thus for every $(u, v) \in U$, both v and uv are in $\langle b \rangle F$. But 2 tells us that for every $(t, s) \in T$, both s and ts are in $\langle b \rangle H^*$. It then follows from 1 that there are no column edges and no symbol edges between T and U.

Claim 2. $\Gamma[U]$ consists of $\frac{l-m}{2}$ disjoint copies of Y_{2k}.

Observe that, when enumerating the vertices in U, every element of $\langle b \rangle F$ occurs exactly twice as a first coordinate and, by 3, exactly twice as a second coordinate. Thus, each vertex in the induced subgraph $\Gamma[U]$ is incident to exactly one row edge and exactly one column edge, so that the row and column edges in $\Gamma[U]$ form a 2-factor (of $\Gamma[U]$). Specifically, they form $l - m$ disjoint $2k$-cycles $\{C_f : f \in F\}$, with each C_f defined by the vertex-sequence

$$(f, f), (f, fb), (bf, fb), (bf, fb^2), \dots, (b^{k-1}f, fb^{k-1}), (b^{k-1}f, f).$$

It follows from the definitions of H^* and F that $\alpha|_F$ is a fixed-point free involution. Thus, to establish Claim 2 it suffices to show that for every $i, j \in [k]$, every $f, h \in F$, and every $\epsilon, \delta \in \{0, 1\}$, the vertices $(b^i f, fb^{i+\epsilon})$ and $(b^j h, hb^{j+\delta})$ are joined by a symbol edge if and only if $j \equiv i + k/2 \pmod{k}$, $h = \alpha(f)$, and $\epsilon = \delta$.

The "if" direction of this equivalence follows directly from the definition of α. For the converse direction we assume

$$b^i f^2 b^{i+\epsilon} = b^j h^2 b^{j+\delta}$$

and, as latin square graphs are loopless, $(b^i f, fb^{i+\epsilon}) \neq (b^j h, hb^{j+\delta})$. It follows from 3 and 4 that

$$b^i f^2 b^{i+\epsilon} \in b^{2i+\epsilon} F \quad \text{and} \quad b^j h^2 b^{j+\delta} \in b^{2j+\delta} F.$$

Thus $\epsilon = \delta$ and $|i - j| \in \{0, k/2\}$.

Now if $i=j$, then $b^i f^{\,2} b^{i+\epsilon} = b^i h^2 b^{i+\epsilon}$ and 4 implies $f = h$, contradicting the fact that $(b^i f, fb^{i+\epsilon}) \neq (b^j h, hb^{j+\delta})$. It follows that j is the unique element of $[k]$ satisfying $j \equiv i + k/2 \pmod{k}$. Thus

$$b^i h^2 b^{i+\epsilon} = b^{i+k/2} f^2 b^{i+\epsilon+k/2} = b^i \alpha(f^2) b^{i+\epsilon},$$

so $h^2 = \alpha(f^{\,2}) = (\alpha(f\,))^2$ and 4 implies $h = \alpha(f)$.

Claim 3. $\Gamma[T]$ is isomorphic to M_{km}

Observe that, when enumerating the vertices in T, every element of $\langle b \rangle H^*$ occurs exactly twice as a first coordinate and, as $\langle b \rangle$ and H^* commute by 2, exactly twice as a second coordinate. Thus, as is the case for $\Gamma[U]$, each vertex in $\Gamma[T]$ is incident to exactly one row edge and exactly one column edge. Unlike in $\Gamma[U]$, the row and column edges of $\Gamma[T]$ form a single cycle of length $2mk$. Indeed, as m is odd and $|b|$ is a power of 2,

$$(h_0, h_0), (h_0, h_1 b), (bh_1, h_1 b), \ldots, (b^{k-1} h_{m-1}, b^{k-1} h_{m-1}), (b^{k-1} h_{m-1}, h_0)$$

is a Hamilton cycle in $\Gamma[T]$ which contains all of $\Gamma[T]$'s row and column edges.

To establish Claim 3 it suffices to show that for every $i, j \in [km]$ and every $\epsilon, \delta \in \{0, 1\}$, the vertices $(b^i h_i, h_{i+\epsilon} b^{i+\epsilon})$ and $(b^j h_j, h_{j+\delta} b^{j+\delta})$ are joined by a symbol edge in $\Gamma[T]$ if and only if $i \equiv j + \frac{k}{2} m \pmod{km}$ and $\epsilon = \delta$.

Indeed if $i \equiv j + \frac{k}{2} m \pmod{km}$, then $i \equiv j \pmod{m}$ and, as m is odd, $i \equiv j + k/2 \pmod{k}$. Together with $\epsilon = \delta$, as well as Lemma 5 and the definition of H^*, this implies

$$b^i h_i h_{i+\epsilon} b^{i+\epsilon} = b^j a h_j h_{j+\delta} a b^{j+\delta} = b^j h_j h_{j+\delta} b^{j+\delta},$$

which establishes the "if" direction of the desired equivalence.

For the converse direction consider $(b^i h_i, h_{i+\epsilon} b^{i+\epsilon}), (b^j h_j, h_{j+\delta} b^{j+\delta}) \in T$ and assume that the group elements defining this pair of distinct vertices satisfy

$$b^i h_i h_{i+\epsilon} b^{i+\epsilon} = b^j h_j h_{j+\delta} b^{j+\delta}.$$

From 2 we see that

$$b^i h_i h_{i+\epsilon} b^{i+\epsilon} \in b^{2i+\epsilon} H^* \quad \text{and} \quad b^j h_j h_{j+\delta} b^{j+\delta} \in b^{2j+\delta} H^*.$$

Thus $\epsilon = \delta$ and $i \equiv j \pmod{k/2}$. Now $b^j \in \{b^i, b^{i+k/2}\}$ and as H^* is pointwise fixed by the automorphism $\alpha : h \mapsto b^{k/2} h b^{k/2}$, both possible values of b^j yield $h_i h_{i+\epsilon} = h_j h_{j+\epsilon}$. Lemma 5 then implies $h_i = h_j$, so $i \equiv j \pmod{m}$.

Suppose $b^j = b^i$, which is equivalent to $i \equiv j \pmod{k}$. As $\gcd(k, m) = 1$ and $i, j \in [km]$, this implies $i = j$, contradicting the fact that $(b^i h_i, h_{i+\epsilon} b^{i+\epsilon})$ and $(b^j h_j, h_{j+\delta} b^{j+\delta})$ are distinct vertices. Therefore $j \equiv i + k/2 \pmod{k}$ and, as $k/2$ and m are coprime, we conclude that $i \equiv j + \frac{k}{2} m \pmod{km}$. □

3 | CONCLUDING REMARKS

The most obvious extension of the present paper is the general case of Conjecture 1. However, a proof of this conjecture would likely differ substantially from the argument presented above. Indeed, latin square graphs are in general not vertex-transitive, calling into question whether general latin square graphs can be shown to possess the sort of "nice" induced subgraphs found in Lemma 3.

There is, however, an extension of Theorem 2 to which the above techniques may be applicable. A partial transversal is *nonextendable* if it is not contained in any larger partial transversal. The following conjecture was noted by Evans [9, p. 470] as a weak version of a conjecture of Keedwell concerning sequenceable groups.

Conjecture 7 (Evans [9]). *For every finite non-Abelian group G, the latin square L(G) possesses a nonextendable near transversal.*

If true, Conjecture 7 cannot be extended to any Abelian groups: an old result of Paige [21] implies that, if G is Abelian, then $L(G)$ possesses either a transversal or a nonextendable near transversal, but it cannot possess both. In contrast, it is known that, for every integer $k \geq 1$, the Cayley table of dihedral group D_{4k+2} of order $4k+2$ possesses both a transversal and a non-extendable near transversal [17].

We have established Conjecture 7 for those non-Abelian groups which do not possess transversals. Perhaps our techniques can be used to find maximal independent sets of size $n-1$ in latin square graphs based upon non-Abelian groups with noncyclic or trivial Sylow 2-subgroups. As far as we know, Conjecture 7 has not been attacked directly. However, many partial results are known due to its connection to sequenceable groups (see eg, [20]).

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