Royal Flush Bonuses

Brian Alspach

Three months ago I received an e-mail asking about the probability of a player winning a "royal flush bonus" in hold'em given that the player must have two of the royal flush cards in the hole. Because the answer to this question is easily determined, let's also consider the corresponding question for Omaha and seven-card stud. The total number of boards for both hold'em and Omaha is $C(52,5) = 2,598,960$. To determine how many allow the royal flush bonus, we observe that there are four choices for the suit and 10 choices for three out of the five royal cards in that suit. The remaining two cards may be any two of the other 47 cards. Thus, there are $4 \cdot 10C(47,2) = 43,240$ boards allowing a royal flush bonus. Dividing by $C(52,5)$ yields a probability of 1,081/64,974 that the board allows a royal flush bonus. Now a given player has $C(47,2) = 1,081$ possible two-card hands. It follows that a given player has a probability of 1/1,081 that she holds the two magic cards giving her a royal flush. Multiplying yields a probability of 1/64,974 that you will win a royal flush bonus in any hand of hold'em. In short, don't count on this bonus to pay your phone bill.

To obtain the probability for Omaha, we observe that a given player has $C(47,4) = 178,365$ possible Omaha hands. The number of those hands containing the two magic cards is $C(45,2) = 990$ since two of her cards are determined and she is choosing two cards from 45 to complete her hand. Hence, the probability is 6/1,081 that a given player has an Omaha hand containing the required two cards. Multipying by the probability of a board allowing a royal flush bonus, we obtain 1/10,829 as the probability an Omaha player will qualify for the royal flush bonus.

To determine the number of seven-card stud hands with a royal flush, note that there are four choices for a royal flush and $C(47,2) = 1,081$ choices for the other two cards. Thus, we see there are 4,324 seven-card hands with royal flushes. Dividing this by $C(52,7)$, the total number of seven-card hands, we obtain a probability of 1/30,940 that a given player has a royal flush in seven-card stud.

The preceding numbers give the probabilities that a *single given player* wins the royal flush bonus in each of hold'em, Omaha, and seven-card stud. Now, let's now look at the bonus from the viewpoint of the cardroom. In other words, let's determine the probability that any player wins the royal flush bonus in each of the three games.

In both hold'em and Omaha it is impossible for more than one player to win the royal flush on a given hand—there will be only one suit on the board that will allow a flush and the winning player must have two of the royal flush cards locked up in the hole. This means the *events* of different players winning the royal flush bonus are *independent*. The significance of independence is that the probability of someone winning the royal flush bonus is simply just the product of the number of players and the probability of a fixed player winning the bonus.

Therefore, if the cardroom is running a hold'em game with n players, the probability of the royal flush bonus occurring is $n/64{,}974$. A casino that deals 10-handed hold'em games expects to have to pay the royal flush bonus about once every 6,500 hands. A similar methodology holds for Omaha. A cardroom dealing 10-handed Omaha is going to pay the royal flush bonus about once every 1,100 hands.

Seven-card stud is not *independent*; that is, it is possible (though highly unlikely) for more than one person to have a royal flush in a given hand. We must, therefore, employ inclusion-exclusion to obtain the exact value. However, we may obtain a close approximation by another method that gives us a value of about 1 in 4,000 for the probability that any player will win the royal flush bonus in an eight-handed seven-card stud game.

Of course, the preceding numbers are affected by the way people actually play hands. The frequency will be diminished because certain hands that would have won the bonus will be folded before they complete a royal flush. For example, a person playing Q-10 of hearts who sees a flop of K-K-4, with one of the kings a heart, and facing heavy betting may fold. The turn and river cards, that would have completed the royal flush, are never dealt. Nevertheless, it is interesting to see the ballpark figures for this promotional reward.