**MR0294594 (45 3664)** 28A15

**Thomson, B. S.**

**Covering systems and derivates in Henstock division spaces.**

*J. London Math. Soc.* (2) **4** (1971), 103–108.

Author's introduction: "The theory of division spaces introduced by R. Henstock [*Linear analysis*, Plenum, New York, 1967; Proc. London Math. Soc. (3) **19** (1969), 509–536; MR0251189 (40 #4420)] in order to simplify and unify certain areas in the theory of integration is particularly well suited to accommodating various parts of the theory of covering systems and differentiation. In this paper we present an introduction to these ideas in a quite general setting. Our approach involves the elegant idea due to Henstock [*Theory of integration*, Butterworths, London, 1963; MR0158047 (28 #1274)] of introducing a further measure on the space, called the inner variation; proving various results on derivates which hold almost everywhere with respect to this measure; and then imposing Vitali conditions to assure that the measure coincides with the original one." *S. J. Taylor*

**MR0280666 (43 6385)** 28.25

**Thomson, B. S.**

**Construction of measures and integrals.**

*Trans. Amer. Math. Soc.* **160** 1971 287–296

Let $\mathfrak{B}$ be a semiring of subsets of a space $T$. A finite subset $\mathfrak{D}$ of $\mathfrak{B} \times T$ whose first elements are pairwise disjoint is a division. Consider a collection $\mathfrak{A}$ of subsets $\mathbf{S}$ of $\mathfrak{B} \times T$ such that $(\varnothing, x)$ is in every $\mathbf{S}$ in $\mathfrak{A}$ for every $x$ in $T$ and $\mathfrak{A}$ is directed downwards by set inclusion. $(T, \mathfrak{A}, \mathfrak{B})$ is termed a division system. With each real valued function $\mu$ on $\mathfrak{B} \times T$ and every $\mathbf{S}$ there is associated a variation $V(\mu, \mathbf{S})$ whose value is the supremum of the sums $\sum |\mu(I, x)|$ for $(I, x)$ in $\mathfrak{D}$, a division contained in $\mathbf{S}$. The infimum of $V(\mu, \mathbf{S})$ for $\mathbf{S}$ in $\mathfrak{A}$ is denoted by $V(\mu, \mathfrak{A})$. Making suitable hypotheses on the division system $(T, \mathfrak{A}, \mathfrak{B})$ the author develops a general theory of measure and integration, using the variation as his principal tool for definition. For example, the outer measure of a subset $X$ of $T$ generated by $\mu$ is $V(\mu, \mathfrak{A}[X])$, where $\mathfrak{A}[X]$ is the collection of $\mathbf{S}[X]$ for $\mathbf{S}$ in $\mathfrak{A}$ consisting of those $(I, x)$ in $\mathbf{S}$ for which $x$ is in $X$. And under suitable restrictions the integral of a non negative real valued function $f$ on $T$ with respect to $\mu$ is $V(f\mu, \mathfrak{A})$. *P. V. Reichelderfer*

**MR0268350 (42 3248)** 28.25

**Thomson, B. S.**

**Constructive definitions for non-absolutely convergent integrals.**

*Proc. London Math. Soc.* (3) **20** 1970 699–716

The author considers a theory of the integral that differs from the usual convergence factor theory. He presents constructive definitions of integrals in developments that are carried forward on the basis of the geometry of the integration process and which rest largely on considerations of topological properties of various Banach spaces.

*W. D. L. Appling*

**MR0259585 (41 4223)** 46.35

**Thomson, B. S.**

**Spaces of conditionally integrable functions.**

*J. London Math. Soc.* (2) **2** 1970 358–360

From the author's introduction: "Let $CL(a,b)$, $D^*(a,b)$ and $D(a,b)$ denote the linear space of functions that are integrable on the interval $[a,b]$ in the Cauchy-Lebesgue, Denjoy-Perron and Denjoy-Hinčin sense, respectively, topologized by the seminorm $f \to \sup\{|\int_a^t f(\xi)\,d\xi|; a \leq t \leq b\}$, where the integral is in the appropriate sense; let $\mathbf{CL}(a,b)$, $\mathbf{D}^*(a,b)$ and $\mathbf{D}(a,b)$ be the associated separated spaces. W. L. C. Sargent [same J. **28** (1953), 438–451; MR0056832 (15,134b)] established several theorems of the Banach-Steinhaus type for these spaces; from a modern point of view it is evident that these results may be interpreted as saying that each of the above spaces is barrelled. In this paper we present a direct proof of this observation."

*B. Yood*