

**MR2202919 (Review)** 26A42 26A24 26A39

**Hagood, John W.** (1-NAZ-MS); **Thomson, Brian S.** (3-SFR)

**Recovering a function from a Dini derivative.**

*Amer. Math. Monthly* **113** (2006), no. 1, 34–46.

The inversion formula

$$F(b) - F(a) = \int_a^b D^+ F(x) dx$$

for a function  $F$  having finite upper right-hand Dini derivative  $D^+ F(x)$  at each  $x \in \mathbb{R}$  is discussed. The authors seek a suitable Riemann-type definition of the integral to obtain this formula without an integrability assumption for  $D^+ F$ . To this purpose, the notion of so-called right full cover (a special case of covering relation due to the second author [B. S. Thomson, *Mem. Amer. Math. Soc.* **93** (1991), no. 452, vi+96 pp.; MR1078198 (92d:26002)]) is introduced. The inversion formula is established for any continuous  $F$ , with the integral being understood as the lower (Henstock-Kurzweil-type) integral defined with respect to right full covers. Clearly, an analogous result holds for any of the remaining three Dini derivatives  $D_+ F$ ,  $D^- F$ ,  $D_- F$ .

Piotr Sworowski (PL-UKW)

#### [References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

1. R. G. Bartle, *A Modern Theory of Integration*, American Mathematical Society, Providence, 2001. MR1817647 (2002d:26001)
2. J. Hagood, The Lebesgue differentiation theorem via nonoverlapping interval covers, *Real Anal. Exch.* **29** (2003–04) 953–956. MR2083830 (2005d:26007)
3. R. Henstock, A Riemann-type integral of Lebesgue power, *Canad. J. Math.* **20** (1968) 79–87. MR0219675 (36 #2754)
4. R. Henstock, *Linear Analysis*, Butterworth, London, 1967. MR0419707 (54 #7725)
5. S. Leader, What is a differential? A new answer from the generalized Riemann integral, this *Monthly* **93** (1986) 348–356. MR0841112 (87e:26002)
6. H. Lebesgue, *Leçons sur l'Intégration*, 3rd ed., Chelsea, New York, 1973; reprint of the 2nd ed., Gauthier-Villars, Paris, 1928.
7. P. Y. Lee and D. S. Zhao, Upper and lower Henstock integrals, *Real Anal. Exchange* **22** (1996–97) 734–739. MR1460984 (98h:26010)

8. S. Saks, *Theory of the Integral*, Dover, New York, 1964; reprint of the 2nd revised ed., G.E. Stechert, Warsaw, 1937. MR0167578 (29 #4850)
9. B. S. Thomson, On full covering properties. *Real Anal. Exchange* **6** (1980–81) 77–93. MR0606543 (82c:26008)