Due: Friday, February 16th (in class)

1. Problem 1-6 from the AMPL book.

2. Problem 2-7 from the AMPL book. Please group all the submission files for this assignment into a single e-mail.

3. For this question, use the digits of your student id, in order: a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9. Consider the linear program:

$$\text{(LP)} \qquad \begin{array}{rc} \max & x_1 + x_2 \\ \text{such that} & a_1x_1 + a_2x_2 \leq a_1 + a_2 + a_3 + 3 \\ & a_4x_1 + a_5x_2 \leq a_4 + a_5 + a_6 + 3 \\ & a_7x_1 + a_8x_2 \leq a_7 + a_8 + a_9 + 3 \\ & x_1, x_2 \geq 0 \end{array}$$

a. Write (LP) in standard equality form (with slacks). Construct a basic feasible solution with $(x_1, x_2) = (0, 0)$.

b. Solve the problem using the simplex method. Which columns form an optimal basis?

c. On a graph, clearly illustrate the feasible region of (LP) in terms of the original variables. Indicate the path taken by the simplex algorithm.

d. Write the dual of the original (inequality) linear program. What is its optimal solution?

4. Vanderbei (5th edition) problem 2.15. (Page 24, be careful about problem renumbering from recent editions.)

5. Vanderbei (5th edition) problem 2.17.

6. Vanderbei (5th edition) problem 2.18.

7. Show that if an LP has an optimal solution and an extreme point, then it has an optimal solution which is an extreme point.

8. Vanderbei (5th edition) problem 4.8.

9. Let $P = \{x \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 1,\ x_1, x_2, x_3, x_4 \geq 0\}$. Describe the edges of P that meet at vertex $(0, 0, 0, 1)$.

10. Give an inequality description of a tetrahedron in $\mathbb{R}^3$ whose projection into the xy plane is a triangle, but whose projection into the xz plane is a quadrilateral. Also list the vertices of the tetrahedron.

11. If $P \subset \mathbb{R}^d$ is the solution to a set of m inequalities, given an upper bound for the number of inequalities need to describe the projection of P to $\mathbb{R}^{d-1}$ (say by collapsing the last co-ordinate).

Please consult me if you have not yet chosen your research paper.