Due: Monday, October 31st (in class)

Reminders

The midterm will take place in class on Friday, October 21st. It will cover material from class up to Wednesday, October 19th.

Math 708 students must select a presentation topic and a date for the presentation. Please consult me if you have not done this.

Reading

Appendix A, Section 2.1 up to, but not including, the subheading *Subadditivity*, Section 2.2 and Section 4.3.

Problems for Math 408 and Math 708

1. For each of the following sets, find a valid inequality cutting off the given fractional point:

   1. $\{(x_1, x_2) \in \mathbb{Z}^2_+ \mid x_1 \leq 5,\ \ x_1 \leq 4x_2\} \qquad (x_1^*, x_2^*, x_3^*) = (5, \frac{5}{4})$.
   2. $\{(x_1, x_2, x_3) \in \mathbb{Z}^3_+ \mid x_1 + x_2 - 2x_3 \leq 0,\ \ x_1, x_2, x_3 \leq 1\} \qquad (x_1^*, x_2^*, x_3^*) = (1, 0, \frac{1}{2})$.
   3. $\{(x_1, x_2, x_3, x_4) \in \mathbb{Z}^4_+ \mid 4x_1 + 8x_2 + 7x_3 + 5x_4 \leq 33\} \qquad (x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, \frac{33}{7}, 0)$.

You should explain how you know that the inequality is valid.

2. Consider the integer programming problem:

$$(IP) \qquad \begin{array}{ll} \text{maximize} & 4x_1 - 6x_2 \\ \text{subject to} & x_1 + x_2 \leq 5, \\ & 2x_1 - 3x_2 \leq 1, \\ & \quad x_1, x_2 \geq 0; \qquad x_1, x_2 \in \mathbb{Z} \end{array}$$

Find the point x^* maximizing the linear programming relaxation of this problem, and find an inequality cutting x^* but no feasible points of (IP).

3. Consider solving a maximization problem by branch-and-bound. Suppose that Figure 1 represents part of the branch-and-bound tree. The value inside each node represents the solution to the associated relaxation of the problem; nodes are coloured blue if the relaxed solution is non-integer, green if it is integer, and red if no relaxed solution exists because the problem is infeasible.

   1. Which node(s) still need to be expanded?
   2. Which node(s) are candidate for the optimal solution?
   3. Suppose that before beginning the search, we obtained an integer feasible solution with value 50. Which node(s) would we no longer have to expand?

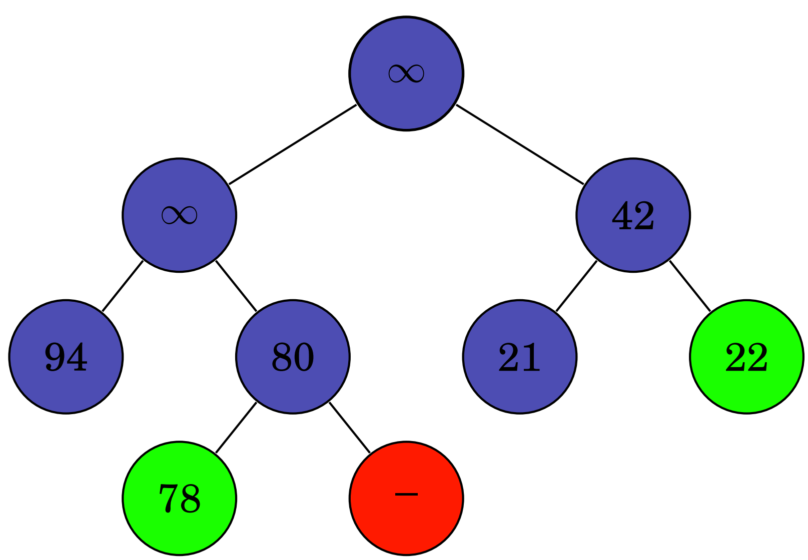

Figure 1: Branch and bound tree for problem 3.

4. Exercise 20.3 (a) and (b) from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

The files used to produce your final solutions should be collected in an electronic archive (e.g. a .zip or .gz file) and e-mailed directly to him at `arafiey` at `sfu.ca`. Use files names of the form `hw3-4-stephen.dat`, replacing "stephen" with your own surname and extensions as appropriate. Make sure to include your final solution as well as the input files.

5. Prove that if a cone is pointed, then it only has one extreme point, namely $\vec{0}$.

Additional Problems for Math 708

6. Textbook Exercise 2.5.

7. Prove that the intersection of any two faces of a polytope P is also a face of P.

8. Consider the problem of finding a maximum stable set of a graph (a maximum set of vertices with no two vertices sharing an edge). Formulate this problem as:

$$\max \sum_{v \in V} x_v \quad \text{subject to} \quad x_{v_1} + x_{v_2} \leq 1 \quad \forall (v_1, v_2) \in E \quad \text{and} \quad x \in \{0,1\}^{|V|}$$

Show that for any complete subgraph (*clique*) W of G, you can obtain the clique inequality $\sum_{v \in W} x_v \leq 1$ by repeatedly applying rounding cuts.