7-Card Stud Low Hands

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Abstract

We enumerate the profiles for low hands in 7-card stud, where an 8 low is required in order for the hand to qualify

We are interested in looking at hands that qualify for low in 7-card stud, where an 8-low or better is required. The total number of 7-card hands is $\binom{52}{7} = 133,784,560$. We may use this number to calculate a variety of probabilities.

The following table contains considerable information about the numbers of

high hand	8-low	7-low	6-low	5-low
straight flush	4,996	5,020	5,020	5,020
flush	519,304	301,340	128,540	28,824
straight	747,980	747,980	747,980	747,980
3-of-a-kind	172,040	70,840	20,240	-
two-pair	771,120	317,520	90,720	-
one pair	5,951,880	2,922,480	962,280	-
high card	5,003,880	3,061,380	1,196,580	-
total	13,171,000	7,426,560	3,151,360	781,824

various low hands in 7-card stud. I believe reading the table is straightforward, but just to make certain here is an example. If we want to know how many 7-card hands there are that are 6-low and have 3-of-a-kind for a high hand, we look across the row corresponding to 3-of-a-kind until we come to the column headed "6-low." We find that there are 20,240 such hands.

If we want to know how many 7-low hands there are, we go to the row corresponding to totals and go until we reach the appropriate column. We find that there are 7,426,560 7-low hands. To get the probability of achieving a 7-low, we divide by 133,784,560 and obtain .0555. This is essentially 1/18. Thus, the odds against being dealt a 7-low in 7-card stud are about 17-to-1.

The rest of this file consists of the derivation of the numbers shown in the above table.

We first make a crude partition of the 7-card hands into hands that qualify for low and those that do not. There are 8 ranks qualifying for low: A, 2, 3, 4, 5, 6, 7, and 8. There are 32 cards of these 8 ranks. The remaining 20 cards do not qualify for low.

Seven distinct low ranks. There are 8 ways to choose 7 distinct low ranks giving us 8 rank sets with 7 low ranks. There are 4 choices of a card for each rank so that each rank set produces $4^7 = 16,384$ hands. Thus, there are

$$8 \cdot 4^7 = 131,072 \tag{1}$$

7-card hands consisting of cards with 7 distinct low ranks.

Three of the rank sets give a 5-low and the other 5 rank sets give a 6-low. Thus, we have

$$3 \cdot 4^7 = 49,152$$

of the hands which are 5-low, and

$$5 \cdot 4^7 = 81,920$$

of the hands which are 6-low.

Six distinct low ranks. There are $\binom{8}{6} = 28$ ways of choosing 6 distinct low ranks. This leaves one extra card for which there are two possibilities. The extra card can be either a card of big rank (9, 10, J, Q or K), or it can pair one of the low cards. We must count these two types separately.

If the extra card has big rank, then we have 28 choices for the set of low ranks and 5 choices for the big rank. This gives us 140 rank sets of this type. We then have 4 choices for each of the 7 ranks giving us

$$140 \cdot 4^7 = 2,293,760$$

7-card hands of this type.

The 140 rank sets break into 15 that are 5-low, 50 that are 6-low, and 75 that are 7-low. This gives us

$$15 \cdot 4^7 = 245,760$$

of the hands that are 5-low,

$$50 \cdot 4^7 = 819,200$$

hands that are 6-low, and

$$75 \cdot 4^7 = 1,228,800$$

hands that are 7-low.

If we have a pair of low rank, then we have 28 possible rank sets. This gives 6 choices for the rank that is paired, 6 choices for a pair of the appropriate rank, and 4 choices for a card of each the other 5 ranks. This gives us

$$28 \cdot 6 \cdot 6 \cdot 4^5 = 1,032,192$$

7-card hands of this type.

The 28 rank sets break into 3 that are 5-low, 10 that are 6-low, and 15 that are 7-low. Multiplying each by $6^2 \cdot 4^5$ produces

$$3 \cdot 6^2 \cdot 4^5 = 110,592$$

hands that are 5-low,

$$10 \cdot 6^2 \cdot 4^5 = 368,640$$

hands that are 6-low, and

$$15 \cdot 6^2 \cdot 4^5 = 552,960$$

hands that are 7-low.

Summing the two numbers above gives us

$$2,293,760 + 1,032,192 = 3,325,952 \tag{2}$$

7-card hands with exactly 6 distinct low ranks.

Five distinct low ranks. There are $\binom{8}{5} = 56$ choices for a set of 5 distinct low ranks. This leaves two cards for which there are several subcases. Both cards may have big ranks, one of the cards may have big rank and the other card pairs one of the low cards, both cards may have low ranks so that the hand has either 3-of-a-kind or two pairs. We do all of these subcases separately.

Because of our interest in breaking the lows into the appropriate values, let's see how the 56 choices of 5 low ranks partition. Exactly 1 of them is 5-low, 5 of them are 6-low, 15 of them are 7-low, and the remaining 35 are 8-low. We shall use these values for this case.

If the two extra cards have big ranks, there are 56 choices for the set of low ranks, $\binom{20}{2} = 190$ choices for the two cards of big ranks, and 4 choices for each card of low rank. We obtain

$$56 \cdot 190 \cdot 4^5 = 10,895,360$$

7-card hands for this subcase.

Now suppose we have one card of big rank and a pair of low rank. We have 5 choices for the rank of the pair, 6 choices for a pair of that rank, 20 choices for the card of big rank, and 4 choices for each card of the other low ranks. This gives us

$$56 \cdot 5 \cdot 6 \cdot 20 \cdot 4^4 = 8,601,600$$

qualifying low hands for this subcase.

If the player has two pairs, there are $\binom{5}{2} = 10$ choices for the two ranks that are paired, 6 choices for pairs of each of those two ranks, and 4 choices for cards of the other three ranks. If the player has 3-of-a-kind, there are 5 choices for the rank of the trips, 4 choices for trips of that rank, and 4 choices for cards of the other four ranks. Combining the calculation into a single expression, we have

$$56[(10 \cdot 6^2 \cdot 4^3) + (5 \cdot 4 \cdot 4^4)] = 1,576,960$$

qualifying hands for this subcase.

Summing the results for the preceding three subcases yields

$$10,895,360 + 8,601,600 + 1,576,960 = 21,073,920 \tag{3}$$

7-card hands with 5 distinct low ranks that qualify for 8-low.

Since all of the numbers used to get the value in (3) were multiples of 56, we can break it down proportionally into the appropriate lows. When we divide by 56, we obtain 376,320. Therefore, there are

$$376,320$$

of them that are 5-low,

$$5 \cdot 376,320 = 1,881,600$$

of them that are 6-low,

$$15 \cdot 376,320 = 5,644,800$$

of them that are 7-low, and

$$35 \cdot 376,320 = 13,171,200$$

of them that are 8-low.

The sum of (1), (2) and (3) is 24,530,944. Dividing this number by the total number of 7-card hands above gives us a probability of .18336 that a player makes a qualifying 8-low in 7-card stud. This is roughly 1/5.5 so that the odds against making a qualifying low are about 4.5-to-1.

Going back through the material, we can sum the various numbers for the different lows. Doing so gives

- 781,824 hands that are 5-low,
- 3,151,360 hands that are 6-low,
- 7,426,560 hands that are 7-low, and
- 13,171,200 hands that are 8-low

which corroborates the values given in the table.

We now want to refine the partition of the low hands further to take into account what high hands some of them hold simultaneously. Hands that qualify for low and also contain a strong high hand are especially powerful in 7-card stud high-low.

Of the 8 rank sets with 7 distinct low ranks, A-2-3-4-6-7-8 and A-2-3-5-6-7-8 do not give the player at least a straight for all 4^7 choices of cards to fill the ranks. However, some of the choices do give the player a flush. Let's now calculate those.

There are 4 ways to choose all 7 cards in the same suit. To obtain 6 cards in the same suit, there are 7 ways to choose which ranks will be suited, there are 4 choices of suit, and 3 choices of suit for the remaining card. This produces 84 choices with 6 suited cards. Finally, there are $\binom{7}{5} = 21$ ways to choose 5 ranks to be suited, 4 choices for the suit, and 3 choices for each of the other two cards. This gives 756 choices producing 5 suited cards. Summing the numbers gives us 844 choices that give the player a flush.

There are $4^7 = 16,384$ choices for the 7 cards. We have just seen that 844 of the choices give the player a flush. This leaves 15,540 choices not giving the player a flush.

The rank sets A-2-3-4-6-7-8 and A-2-3-5-6-7-8 then lead to 1,688 flushes and 31,080 hands that are simply ace-high as high hands. The other 6 rank sets also have 844 flushes each, but the remaining 15,540 choices give the player a straight. However, we still need to adjust the numbers because some of the choices producing flushes actually produce straight flushes The rest of the calculations for 7 distinct low ranks serve to count how many choices produce straight flushes.

For each of the remaining 6 rank sets, if all 7 cards are suited, then the player certainly has a straight flush. There are 4 such choices for each rank set. There are 84 choices for each of them with 6 suited cards. For A-2-3-4-5-6-7 and 2-3-4-5-6-7-8, 48 of the choices are straight flushes and 36 are flushes. For A-3-4-5-6-7-8 and A-2-3-4-5-6-8, 36 of the choices are straight flushes and 48 are flushes. For the other two, we have 24 straight flushes and 60 flushes. Altogether there are 240 straight flushes when choosing 6 or 7 suited cards.

Two of the rank sets have 3 ways of choosing 5 consecutive ranks, two have 2 ways of choosing 5 consecutive ranks, and two have a unique way of choosing 5 consecutive ranks. Each choice of 5 consecutive ranks yields 36 straight flushes. This gives us another $12 \cdot 36 = 432$ straight flushes.

In summary, for the 8 rank sets with 7 distinct low ranks, we have

- 672 straight flushes,
- 6,080 flushes,
- 93,240 straights, and
- 31,080 high card hands.

Of the 672 straight flushes, 336 belong to hands that are 5-low and 336 belong to hands that are 6-low. Of the 6,080 flushes, 2,196 belong to hands that are 5-low and 3,884 belong to hands that are 6-low. The hands that are straights divide evenly between 5-low hands and 6-low hands, so that 46,620 are 5-low and 46,620 are 6-low. The 31,080 high card hands all are 6-low.

We now move to the rank sets with 6 distinct low ranks. We saw earlier that there are $5 \cdot 28 = 140$ rank sets, with 7 distinct ranks, involved. The only types of strong high hands that can be realized for these rank sets are straights and flushes. Each of the ranks sets gives rise to 844 choices producing flushes.

Some of these flushes are, in fact, straight flushes and some of the rank sets do not contain straights.

The first separation we perform is to determine which of the 140 rank sets allow straights. The only way the big rank can play a role in straights is if the big rank is 9. Otherwise, only the six low ranks may be used to form straights. The only choice that gives 7 consecutive ranks is 3 through 9. For 6 consecutive ranks, if 9 is included, then we have 4 through 9 with the missing rank being A or 2. For 6 consecutive ranks, if 9 is not included, then we have one of A through 6, or 2 through 7, or 3 through 8. For the first two, we may use any of the 5 big ranks, and for the last we may use any big rank other than 9. This gives us 16 rank sets with 6 consecutive ranks and 1 rank set with 7 consecutive ranks.

It takes a little more effort to count the rank sets with precisely 5 consecutive ranks. If the 5 consecutive ranks are A through 5, then the other low rank is 7 or 8 and there are 5 choices for big rank giving us 10 rank sets. If the 5 consecutive ranks are 2 through 6, then the only choice for the last low rank is 8, with 5 choices for the big rank. This is another 5 rank sets. If the 5 consecutive ranks are 3 through 7, then A is the only choice for the missing low rank, with 5 choices for the big rank. We have another 5 rank sets. If the 5 consecutive ranks are 4 through 8, we may choose either A or 2 for a low rank, but cannot choose rank 9 for a big rank. We then obtain another 8 rank sets. Finally, if the 5 consecutive ranks are 5 through 9, then we may choose any 2 from {A,2,3} giving us another 3 rank sets. Summing all these numbers yields 31 rank sets with exactly 5 consecutive ranks.

The remaining 92 rank sets do not contain a straight. For each of these rank sets, we have 844 flushes and 15,540 high card hands upon making all possible choices for the suits.

The one rank set with 7 consecutive ranks gives rise to 160 straight flushes and 684 flushes. The rest of the choices, of course, give the player a straight. Note that these hands are 7-low.

For the rank sets with 6 consecutive ranks, 112 of the choices lead to straight flushes leaving 732 choices that are ordinary flushes. Of these rank sets, 5 are 5-low, 5 are 6-low, and 6 are 7-low.

For the rank sets with 5 consecutive ranks, 64 choices result in straight flushes leaving 780 choices that are flushes. Of the 31 rank sets, 10 are 5-low, 10 are 6-low, and 11 are 7-low. If we perform the appropriate arithmetic based on the numbers of rank sets of the different types, we obtain

- 3,936 straight flushes,
- 114,224 flushes,
- 745,920 straights, and
- 1,429,680 high card hands

for the rank sets with 7 distinct ranks of which 6 are low ranks.

Breaking them up according to the value of the lows, gives 1,200 5-low hands with straight flushes, another 1,200 6-low straight flushes, and 1,536 7-low hands with straight flushes. We also can break up the straights because the rank sets discussed up to this point are the only ones allowing straights. There are 233,100 5-low hands with straights, 233,100 6-low hands with straights, and 279,720 7-low hands with straights.

For flushes and high card hands, we have to consider the 92 rank sets that do not contain a straight. It is not difficult to check that 35 of them are 6-low and 57 of them are 7-low. This then gives us 11,460 hands that are flushes and 5-low, another 41,000 hands that are flushes and 6-low, and 61,764 hands that are flushes and 7-low. Finally, there are 543,900 high card hands that are 6-low and 885,780 high card hands that are 7-low.

We now move to the low hands with 6 distinct low ranks and a pair. Hence, all of these hands have at least a pair as a high hand. We observed earlier that there are 28 possible rank sets. For each rank set, we then have 6 choices for which rank is paired, 6 choices for a pair of that rank and 4 choices for each of the other 5 ranks. Thus, each rank set produces 36,864 possible 7-card hands. We first determine how many of the hands have a flush for each rank set.

Given the rank set, we have 6 choices of the rank for the pair. Once the rank has been chosen, there are 6 choices for the pair. Once the pair has been chosen there are 3 different possible kinds of choices that give flushes. If there are 6 suited cards, then there are only 2 choices for the suits of the other cards. If there are 5 suited cards and the suit does not agree with one of the suits of the pair, then there are only 2 choices for the suit of the other 5 cards. If the suit does agree with one of the suits of the pair, then there are 2 choices for the suit, 5 choices for which of the other 5 cards does not get assigned this suit, and 3 choices for the suit of the contrary card. Altogether, we obtain

$$6^2(2 + 2 + (5 \cdot 2 \cdot 3)) = 1,224$$

choices for the rank set that give a flush.

Thus, for each of the rank sets we have 1,224 choices giving us flushes, and 35,640 choices not giving a flush. However, some of the rank sets already include straights so that some of the flushes become straight flushes, as well as the straight being better than a pair.

Since we have 6 ranks present, it is easy to see that there are exactly 3 rank sets with 6 consecutive ranks. One of them is a 5-low, one of them is a 6-low, and the remaining one is a 7-low. Let's now determine the number of straight flushes for these rank sets. If the pair has either of the ranks on one of the two ends of the 6 consecutive ranks, then there are 2 choices for the rank followed by 6 choices of the pair. If the 5 consecutive ranks not in the pair are in the same suit, we have a straight flush and there are 4 choices for the suit. If those 5 consecutive ranks are not in the same suit, then the pair belongs to the straight flush. This implies there are 2 choices for the suit and 3 choices for the card at the other end of the sequence of ranks. This gives $2 \cdot 6(4 + (2 \cdot 3)) = 120$ straight flushes. Using similar reasoning, we find that when the pair is one of the internal ranks, then there are 336 straight flushes.

Altogether, we have 456 straight flushes, 768 flushes and 35,640 straights. We get this contribution once for a 5-low, a 6-low and a 7-low.

There are 6 rank sets with 5 consecutive ranks. Of these, 2 are 5-low, 2 are 6-low, and 2 are 7-low. Going through an argument like we did above, we find that each of these rank sets produces 264 straight flushes and 960 flushes.

The remaining 19 rank sets do not have 5 consecutive ranks. Their lows are 6-low for 7 of them, and 7-low for 12 of them. Doing the appropriate arithmetic leads to

- 2,952 hands that are straight flushes,
- 31,320 hands that are flushes,
- 320,760 hands that are straights, and
- 677,160 hands that are one pair.

Now we break up the preceding numbers according to the low values of the hands. There are 984 straight flushes for each of 5-low, 6-low and 7-low hands. There are 2,688 flushes for hands that are 5-low, there are 11,256 flushes for hands that are 6-low, and there are 17,376 flushes for hands that are 7-low. There are 106,920 straights for each of 5-low, 6-low and 7-low hands. Finally, there are 249,480 hands that are 6-low and a pair for high, and 427,680 hands that are 7-low and a pair for high.

There are $\binom{8}{5} = 56$ ways of choosing 5 distinct low ranks. There are 10 ways of choosing 2 distinct big ranks so that we have 560 rank sets with 5 low ranks and 2 big ranks. Each rank set again produces $4^7 = 16,384$ different hands of which 844 are flushes.

The rank set {4,5,6,7,8,9,10} is the only one with 7 consecutive ranks. It is an 8-low and produces 160 straight flushes, 684 flushes and 15,540 straights.

The 6 consecutive ranks 5,6,7,8,9,10 together with 1 chosen from A,2,3 fits into our form. Also, the 6 consecutive ranks 4,5,6,7,8,9 together with one chosen from J,Q,K fits into our form. These 6 rank sets all are 8-lows and produce 112 straight flushes, 732 flushes, and 15,540 straights.

There are a variety of ways we can have 5 consecutive ranks. The ranks 6,7,8,9,10 together with 2 chosen from A,2,3,4 give us 6 rank sets. Each of these is an 8-low. The ranks 5,6,7,8,9 together with 1 from A,2,3 and 1 from J,Q,K give us another 9. These also are 8-low. The ranks 4,5,6,7,8 together with 2 chosen from 10,J,Q,K give us 6 more. These too are 8-low.

There are 3 more ways we can have 5 consecutive low ranks together with 2 chosen from 9,10,J,Q,K giving us another 30. Altogether we have 51 rank sets, where 10 are 5-low, 10 are 6-low, 10 are 7-low, and 21 are 8-low.

All of the preceding sets have the property that they generate 64 straight flushes, 780 flushes, and 15,540 straights.

We are left with 502 rank sets not containing a straight. One can show fairly easily that 40 of these rank sets are 6-low, 140 are 7-low, and 322 are 8-low.

Further, as seen earlier, 844 of the hands they generate are flushes and the rest are high card.

Performing the arithmetic from the preceding discussion leads to

- 4,096 hands that are straight flushes,
- 468,544 hands that are flushes,
- 901,320 hands that are straights, and
- 7,801,080 hands that are high card.

We now partition the preceding hands according to their low values. There are 640 straight flush hands that are 5-low, 640 straight flush hands that are 6-low, 640 straight flush hands that are 7-low, and 2,176 hands that are 8-low. There are 7,800 hands that are flushes that are 5-low, 41,560 hands that are flushes that are 6-low, 125,960 hands that are flushes that are 7-low, and 293,224 hands that are flushes that are 8-low. There are 155,400 hands that are straights that are 5-low, 155,400 hands that are straights that are 6-low, 155,400 hands that are straights that are 7-low, and 435,120 hands that are straights that are 8-low. Finally, the high card hands divide into 621,600 that are 6-low, another 2,175,600 that are 7-low, and 5,003,880 that are 8-low.

The next type of rank set we consider is one with 5 low ranks and 1 big rank with a pair of that rank. Because of the 5 choices for the big rank, there are 280 rank sets of this form. There are 6 choices for the pair of big rank and 4^5 choices for the cards of low ranks giving $6 \cdot 4^5 = 6,144$ hands for each rank set. It is not difficult to verify that 204 of the hands are flushes.

The ranks 4,5,6,7,8,9 form the only rank set with 6 consecutive ranks. It is an 8-low and gives rise to 60 straight flushes, 144 flushes, and 5,940 straights. There are then 22 rank sets with 5 consecutive ranks. Of these, 7 are 8-low, 5 are 7-low, 5 are 6-low, and 5 are 5-low. Each gives rise to 48 straight flushes, 156 flushes, and 5,940 straights.

This leaves 257 rank sets that do not contain a straight. Each of them has 204 choices leading to flushes and 5,940 choices with a pair as the high hand. Of the 257 rank sets, 20 are 6-low, 70 are 7-low, and 167 are 8-low.

We now can perform the usual arithmetic to obtain

- 1,116 hands that are straight flushes,
- 56,004 hands that are flushes,
- 136,620 hands that are straights, and
- 1,526,580 hands that are a single pair.

There are 240 straight flushes for each of 5-low, 6-low and 7-low, with the remaining 396 straight flushes being 8-low. There are 780 flush hands that are 5-low, there are 4,860 flush hands that are 6-low, there are 15,060 flush hands that are 7-low, and there are 35,304 flush hands that are 8-low. There are

29,700 hands that are straights for each of 5-low, 6-low, and 7-low, while there are 47,520 hands that are straights and 8-low. Finally, there are 118,800 hands that are one pair and 6-low, 415,800 hands that are one pair and 7-low, and 991,980 hands that are one pair and 8-low.

We now move to the subcase of 5 distinct low ranks and 1 big rank which implies there is a pair of low rank. There are 280 rank sets as in the preceding subcase. There are 5 choices for the rank of the pair, 6 choices for the pair, and 4 choices for each of the other ranks. This gives $5 \cdot 6 \cdot 4^5 = 30,720$ hands for each rank set. The number of choices leading to flushes is $5 \cdot 6(2+2+(2 \cdot 5 \cdot 3) = 1,020$.

There is exactly 1 rank set with 6 consecutive ranks, namely, {4,5,6,7,8,9}. If any of the 4 internal ranks are paired, there are $4 \cdot 6(2+(2 \cdot 3)+(2 \cdot 3)) = 336$ choices giving straight flushes. If the pair has rank 4, then there are $6(2+2+(2 \cdot 3)) = 60$ choices that are straight flushes. This yields 396 straight flushes, 624 flushes, and 29,700 straights for this rank set. This set gives hands that are 8-low.

There are 3 rank sets with 5 consecutive ranks that include a 9. All of these rank sets give hands that are 8-low. Doing an analysis similar to what we just did gives 216 straight flushes, 804 flushes, and 29,700 straights for each of the rank sets.

There are 5 rank sets for each of the 5-low, 6-low and 7-low rank sets with 5 consecutive low ranks. There are only 4 rank sets for the 8-low case because we cannot use rank 9. For all of these rank sets, there are 240 straight flushes, 780 flushes, and 29,700 straights.

We have 257 rank sets without 5 consecutive ranks. Each of them contributes 1,020 hands that are flushes and 29,700 hands that are pairs. These rank sets partition into 20 that are 6-low, 70 that are 7-low, and 167 that are 8-low.

Performing the arithmetic for this subcase gives

- 5,604 hands that are straight flushes,
- 279,996 hands that are flushes,
- 683,100 hands that are straights, and
- 7,632,900 hands that are one pair.

We now want to break the preceding hands into their low values. There are 1,200 straight flush hands that are 5-low, 1,200 straight flush hands that are 6-low, 1,200 straight flush hands that are 7-low, and 2,004 straight flush hands that are 8-low. There are 3,900 flush hands that are 5-low, 24,300 flush hands that are 6-low, 75,300 flush hands that are 7-low, and 176,496 flush hands that are 8-low. There are 148,500 hands that are straights and 5-low, 148,500 hands that are straights and 6-low, 148,500 hands that are straights and 7-low, and 237,600 hands that are straights and 8-low. Finally, there are 594,000 hands that are 6-low and have just a pair, 2,079,000 hands that are 7-low and have a pair, and 4,959,900 hands that are 8-low and have a pair.

We consider rank sets with just 5 low ranks. One subcase is that we have 3-of-a-kind in the hand. There are 5 choices for the rank of the trips and 4 choices for all the ranks. Thus, each rank set produces $5 \cdot 4^5 = 5,120$ hands. It is easy to see that 60 of the choices give the player a flush.

There are 4 rank sets with 5 consecutive ranks; one each for 5-low, 6-low, 7-low, and 8-low. Each of these gives 60 straight flushes and 5,060 straights.

The other 52 rank sets give us 60 flushes and 5,060 3-of-a-kind hands. Of these 52 rank sets, 4 are 6-low, 14 are 7-low, and 34 are 8-low.

Performing the appropriate arithmetic gives us

- 240 hands that are straight flushes,
- 3,120 hands that are flushes,
- 20,240 hands that are straights, and
- 263,120 hands that are 3-of-a-kind.

We partition the hands according to their low values and obtain 60 straight flushes for each of 5-low, 6-low, 7-low, and 8-low. There are 240 hands that are flushes and 6-low, 840 hands that are flushes and 7-low, and 2,040 hands that are flushes and 8-low. There are 5,060 hands that are straights for each of 5-low, 6-low, 7-low, and 8-low. Finally, there are 20,240 hands that are trips and 6-low, 70,840 hands that are trips and 7-low, and 172,040 hands that are trips and 8-low.

The last situation we consider is 5 distinct low ranks and two-pair. The number of rank sets is still 56, but now each rank set gives rise $10 \cdot 6^2 \cdot 4^3 = 23,040$ hands. There are 360 of these hands that are flushes. The 4 rank sets of 5 consecutive ranks each have 360 straight flushes and 22,680 straights. There is one for each of 5-low, 6-low, 7-low, and 8-low.

The other 52 rank sets give 360 flushes and 22,680 two-pair hands each. These again break into 4 rank sets for 6-low, 14 for 7-low, and 34 for 8-low.

The appropriate arithmetic leads to

- 1,440 hands that are straight flushes,
- 18,720 hands that are flushes,
- 90,720 hands that are straights, and
- 1,179,360 hands that are two-pair.

We now partition the preceding hands according to the lows they make. There are 360 straight flush hands for each of 5-low, 6-low, 7-low, and 8-low. There are 1,440 hands that are flushes and 6-low, 5,040 hands that are flushes and 7-low, and 12,240 hands that are flushes and 8-low. There are 22,680 hands that are straights for each of 5-low, 6-low, 7-low, and 8-low. There are 90,720 hands that are two-pair and 6-low, 317,520 hands that are two-pair and 7-low, and 771,120 hands that are two-pair and 8-low.

This completes the information that is required for completing the entries in the table given at the beginning of the file.