Flushing Boards

Brian Alspach

17 November 1998

Abstract

We determine the probability of a board occurring which allows the possibility of a flush where a board is 5 cards from a standard deck of 52 cards. The result applies to both hold'em and Omaha when no further knowledge about players' hands is available.

In hold'em or Omaha, where 5 cards are displayed in the middle for everyone to use in her or his hand, we refer to the 5 cards in the middle as the *board*. In order for some player to be able to make a flush, there must be at least 3 cards in the same suit in the board. We shall call such a board a *flushing* board. We are interested in determining the probability of having a flushing board in the case none of the players' cards are known.

The total number of possible boards is

$$\binom{52}{5} = \frac{52!}{5!47!} = 2,598,960.$$

Let's express the *type* of suit distribution of the board as a vector with 4 coordinates. The possible types are (0,0,0,5), (0,0,1,4), (0,0,2,3), (0,1,1,3), (0,1,2,2), and (1,1,1,2). For example, the type (0,0,0,5) indicates the board is made up of 5 cards in the same suit. It is easy to see the first 4 types describe flushing boards and the last 2 types describe boards which are not flushing.

The type (0,0,0,5). There are 4 choices for the suit in the board and $\binom{13}{5} = 1,287$ choices for the 5 cards of that suit. This yields $4 \cdot 1,287 = 5,148$ boards of type (0,0,0,5).

The type (0,0,1,4). There are 4 choices for the suit with 4 cards in the board, 3 choices for the remaining suit, $\binom{13}{4} = 715$ choices for the 4 cards of the one suit, and 13 choices for the card in the other suit. This yields $4 \cdot 3 \cdot 715 \cdot 13 = 111,540$ boards whose type is (0,0,1,4).

The type (0,0,2,3). There are 4 choices for the suit with 3 cards, 3 choices for the suit with 2 cards, $\binom{13}{3} = 286$ choices for the 3 cards of the one suit, and $\binom{13}{2} = 78$ choices for the 2 cards of the other suit. This produces $4 \cdot 3 \cdot 286 \cdot 78 = 267,696$ boards of type (0,0,2,3).

The type (0,1,1,3). There are 4 choices for the suit with 3 cards, 3 choices for the 2 remaining suits, $\binom{13}{3} = 286$ choices for the 3 cards of the one suit, and 13 choices for each of the cards of the other 2 suits. This gives us $4 \cdot 3 \cdot 286 \cdot 13^2 = 580,008$ boards of type (0,1,1,3).

The type (0,1,2,2). There are 4 choices for the suit with 1 card, $\binom{3}{2} = 3$ choices for the suits with 2 cards apiece, 13 choices for the card from the first suit, and $\binom{13}{2} = 78$ choices for the 2 cards of each suit. This produces $4 \cdot 3 \cdot 13 \cdot 78^2 = 949,104$ boards of type (0,1,2,2).

The type (1,1,1,2). There are 4 choices for the suit with 2 cards of that suit, $\binom{13}{2} = 78$ choices for the card of that suit, and 13 choices for each of the other cards. This yields $4 \cdot 78 \cdot 13^3 = 685,464$ boards of type (1,1,1,2).

Adding the numbers of boards of the six types yields 2,598,960 boards as it should.

Boards of types (0,0,0,5), (0,0,1,4), (0,0,2,3), and (0,1,1,3) are flushing boards. The number of boards of these types is the sum

$$5,148 + 111,540 + 267,696 + 580,008 = 964,392.$$

This means the probability of a flushing board occurring given no further information is

$$\frac{964,392}{2,598,960} = 0.371.$$

So a flushing board occurs at the rate of about 3 out of every 8 hands.