Is There a Bigger Flush Out There?

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This is the fourth and last article dealing with probabilities of random hands achieving flushes in hold'em and Omaha when the board allows flushes. We have saved the best to last in that we now consider the probabilities that a player holding a flush is facing a bigger flush under varying conditions. All these values are derived using inclusion-exclusion, a technique introduced in the second of the four articles. We avoid details and simply present the information in two tables. The rest of the article deals with how to read the tables and evaluating the information.

L	(0,5)	(1,5)	(1,4)	(2,5)	(2,4)	(2,3)
0	0	0	0	0	0	0
1	.4	.4	.4	.4	.4	.06364
2	.6455	.6455	.6455	.6455	.6455	.11577
3	.7939	.7939	.7939	.7939	.7939	.15764
4	.8822	.8822	.8822	.8822	.8822	.19021
5	.9339	.9339	.9339	.9339	.9339	.21418
6	.9637	.9637	.9637	.9637	.9637	.23001
7	.9804	.9804	.9804	-	.9804	.23792
8	.9897	-	.9897 -	-	-	.23792

HOLD'EM

The table above is for hold'em. Let's make the context completely clear. For the sake of simplicity, we refer to hearts as the flush suit, but all the information equally pertains to any suit. The column headed with L gives the number of bigger hearts among the unseen hearts. The remaining columns are headed with a pair (m,n). The first number in the pair is the number of hearts a fixed player X has in her hand, and the second number in the pair is the number of hearts on board. For example, if we are looking in the column headed by (2,4), then X has two hearts in her hand with four hearts on board.

The entries in the table then give the probability, for the given conditions, that out of nine random hands there is one or more with a bigger flush. An example should clarify the situation. Suppose X has the eight of hearts in her hand and no other hearts, and the board has the 5-9-Q-K of hearts and no other hearts. She has a flush, but among the eight unseen hearts, anyone holding an A, J, or 10 of hearts has a bigger flush. Thus, $L = 3$. In the table you look down the column headed (1,4) until you reach the entry corresponding to the row for $L = 3$. The entry you find is .7939. This is the probability that one or more hands, from nine random two-card hands, has a bigger flush. This is approximately four out of five times.

The next table is a similar table for Omaha. There are some slight differences

Bigger Cards	Unseen Suited Cards				
L	4	5	6	7	8
0	0	0	0	0	0
1	.17072	.22199	.27056	.31651	.3599
2	.27465	.36426	.44353	.5135	.57513
3	.32518	.45018	.55292	.63712	.70594
4	.32518	.49184	.61838	.7141	.78627
5	-	.49184	.65001	.75968	.83542
6	-	-	.65001	.78162	.86414
7,8	-	-	-	.78162	.8779

Omaha

from the hold'em table. Because of the rule that a player must use precisely two cards from her hand, the distribution of hearts between X and the board is irrelevant, other than the fact X must have at least two hearts to have a flush. Consequently, all that is important is the number of unseen hearts and the number of those giving a bigger flush, which is reflected in the column headings for the table.

For example, if X has the 3-9-10 of hearts with a non-heart, and the board has A-4-8-Q of hearts with a non-heart, then the number of unseen hearts is 6 and $L = 2$. We see from the table that there is a probability of .44353 that upon dealing nine random Omaha hands under these conditions, one or two players will have a bigger flush.

The numbers in the tables ignore straight flushes. A natural question is how much distortion arises from ignoring straight flushes. In the case of hold'em, when there are four or five hearts on board and a single small card not already counted in L making a straight flush, add this card to the count for L. For example, if the board has the 4-6-7-8 of hearts, then let the 5 of hearts contribute to L. In this way, there is no distortion.

Some distortion does arise for two-card combinations that do not already contribute to L. A board requiring two particular hearts to make a straight flush, leads to a probability of 1/110 that one of nine random hold'em hands has the two cards, or a probability of approximately 1/35 that one of nine random Omaha hands has the two cards. This strongly suggests how to adjust the table entries for straight flushes.