Online bad-beat jackpots: Part I

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As I write this article, I am in the midst of a fascinating e-mail correspondence with Jeff Kitchen, a lawyer and poker enthusiast, about online bad-beat jackpots. Our continuing electronic exchange is the motivation for this, and my next article.

I ran into Bib Ladder, two granddaughters in tow, on my recent trip to Vancouver. The old gent takes more pleasure from these offspring than the many poker sessions in which he participates. After exchanging pleasantries, Bib asked me, "Professor, at what point does it become profitable to play at an online hold'em bad-beat jackpot table given that it is raking fifty cents per hand to fund the jackpot?" In other words, Bib wanted to know: 'When is the fifty cent cost per hand less than what one expects to win per hand?'

I told Bib that I would derive a numerical answer to his question, and the number itself will be of interest to some readers. But what I find far more interesting, is the journey to reach the number. There is no deep mathematics involved, but there are several junctures at which assumptions must be made. I want to discuss these assumptions carefully and try to assess the impact of the assumptions on the number I derive.

My first step is the derivation of the probability that a 10-handed hold'em game produces a bad-beat jackpot given that no player folds a hand until it is impossible for that hand to qualify for a bad-beat jackpot. In other words, if a player holds a hand such as 2-5 of hearts, the player sees the flop because the hand could develop into a straight flush. If no ace, trey, four or six of hearts comes on the flop, the player may fold the hand following the flop. Otherwise, the player sees the turn and so on.

I now determine the preceding probability exactly. Since I don't care which players qualify for the jackpot, I am interested in the total number of possible semideals in 10-handed hold'em. There are $C(52,5) = 2,598,960$ ways to choose the board. There are then $C(47,20) = 9,762,479,679,106$ ways to choose 20 cards to form the 10 hands. Given 20 cards, there are $19!! = 654,729,075$ ways to divide the cards into 10 hands of two cards each. Multiplying these three numbers gives us the total number of semideals for a 10-handed hold'em game. Because it is such a magnificently large number, here it is on display:

$$16,611,978,703,557,549,675,134,772,000.$$

All I need to do now is count the number of semideals that qualify for the bad-beat jackpot and divide by the preceding number. Thus, I now need the rules that determine whether a hand qualifies. I am going to use the rules posted at PokerParty.com. Essentially, the rules stipulate that quad 8s or better must be beaten, the player must use her best hand, and both hole cards must form her best hand. I still have one interpretive question about their rules, but unfortunately no one from PokerParty has yet answered my query. Here is my question and my interpretation.

Suppose one player has K-10, another player has J-9 of hearts, and the board has 10-10-10-Q-K, where three of them are hearts. The player with J-9 of hearts has a straight flush, and the other player's best hand is quad 10s with a K kicker. I am going to interpret the hand with quad 10s as qualifying even though her kicker in the hole ties the kicker on the board. The boards that allow a bad-beat jackpot may be classified as follows: full house on board; trips on board with two other cards that allow a straight flush; two-pair on board; one pair on board with three other cards that allow a straight flush; or five distinct ranks on board that allow two players to hold simultaneous straight flushes.

All I need to do now is count the number of semideals for each of the preceding types of boards that produce two or more players with qualifying hands. I shall not provide the details for this straightforward, though tedious, computation. Details are available at my website (http://www.math.sfu.ca/ alspach). The probability is very close to 1/155,000.

After working out this value I said to Bib, "The probability is very small; 1/154,997 to be precise. However, this is the last precise statement I can make as we move towards a single number in answer to your original question. From now on, estimates and assumptions come into play."

Next month we shall discuss the assumptions we make to produce an answer.